# Dirac's æther in curved spacetime

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**ABSTRACT**

Proca's equations for two types of fields in a Dirac's æther with electric conductivity $\sigma$ are solved exactly. The Proca electromagnetic fields are assumed with cylindrical symmetry. The background is a static, curved spacetime whose spatial section is homogeneous and has the topology of either the three-sphere $S^3$ or the projective three-space $P^3$. Simple relations between the range of Proca field $\lambda$, the Universe radius $R$, the limit of photon rest mass $m_\gamma$ and the conductivity $\sigma$ are written down.

**Key words:** Dirac's æther, Proca Field, curved spacetime, three-sphere, projective three-space.

## INTRODUCTION

The possibility of a nonzero electric conductivity $\sigma$ in cosmic scale (Dirac's æther) has been considered by several authors and in various contexts: Vigier (Vigier 1990), e.g., showed that introducing $\sigma > 0$ in the vacuum is equivalent to attributing a nonzero mass $m_\gamma > 0$ to the photon. Further study of the relation between $\sigma$ and $m_\gamma$ was performed by Kar, Sinha and Roy (Kar *et al.* 1993), who also discussed possible astrophysical consequences of having nonzero $m_\gamma$. More recently, Ahonen and Enqvist (Ahonen & Enqvist 1996) studied the electric conductivity in the hot plasma of the early universe.

In this paper we study the time evolution of an electromagnetic field with $m_\gamma > 0$; in the background we assume a curved spacetime together with a constant conductivity $\sigma > 0$. In the next section we present the three existing classes of exact solutions for the field; they depend on the relative values of $\sigma$, $m_\gamma$ and the curvature of spacetime as given by a constant radius $R$. In the last section we describe in some detail a set of solutions in which the quantity $\mathbf{E}^2 + c^2\,\mathbf{B}^2$ is homogeneous throughout the spacelike hypersurfaces $t = const$.

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**EQUATIONS AND SOLUTIONS**

In the static elliptic spacetime we use the cylindrical Schrödinger coordinates $x^\mu = (ct; \rho, \phi, \zeta)$ and write the line element

$$ds^2 = c^2 dt^2 - R^2(d\rho^2 + \sin^2 \rho d\phi^2 + \cos^2 \rho d\zeta^2) \ , \tag{1}$$

where $R = const$ is the characteristic radius of the three-geometry.

We assume a nonstatic four-potential with cylindrical symmetry

$$\Phi^\mu (0; 0, cf(t), 0) \ , \tag{2}$$

where $f(t)$ is a function to be determined from the field equations; clearly $\Phi^\mu$ satisfies the Lorentz gauge, $\partial_\mu[(-g)^{1/2}\Phi^\mu] = 0$. The only surviving independent components of $F_{\mu\nu} = \partial_\mu \Phi_\nu - \partial_\nu \Phi_\mu$ are then

$$F_0^2 = \dot{f} \ , \qquad F_1^2 = 2cf \cot \rho \ , \tag{3}$$

where the overdot means the time $t$ derivative. In the orthonormal basis the nonvanishing components of the **E** and **B** fields are

$$E_\phi = -R\dot{f} \sin \rho \ , \qquad B_\zeta = 2f \cos \rho \ . \tag{4}$$

Proca equations in a conducting medium are

$$F^{\mu\nu}_{\ ;\mu} + (\kappa/\lambda^2)\Phi^\nu = (\sigma/c)u_\alpha F^{\nu\alpha} \ , \tag{5}$$

where $\sigma > 0$ is the electric displacement conductivity, $u_\alpha = \delta^0_\alpha$ is the four-velocity of the observer, $\lambda$ is the range of the Proca field, and $\kappa = \pm 1$ accounts for two different categories of field. For $\nu = 2$ eq.(5) gives

$$\ddot{f} + 2\Gamma \dot{f} + \gamma f = 0 \ , \tag{6}$$

where

$$\Gamma = \sigma/2 \ , \qquad \gamma = 4c^2/R^2 + \kappa c^2/\lambda^2 \ . \tag{7}$$

Three classes of solutions of (6) exist, depending on the relative values of the constants $\Gamma$ (nonnegative) and $\gamma$ (arbitrary); see Table I, where $C_1$ and $C^2$ are integration constants.

Solutions in which the field energy is homogeneously distributed in three-space are of particular interest. From eqs.(4) we find that the quantity $\Delta = E_\phi^2 + c^2 B_\zeta^2$ is independent of $\rho$ only when $R^2 \dot{f}^2 = 4c^2 f^2$, which implies that $f(x) \propto \exp(2\epsilon ct/R)$, with $\epsilon = \pm 1$. Three non-equivalent sets of solutions with $\partial\Delta/\partial\rho = 0$ are discussed in the next section, and constraining relations among the quantities $\{\sigma, \lambda, R, c, \kappa, \epsilon\}$ in each set are given.

**TABLE I**

**Potential functions.**

| Classes | Exact solution of (6) |
|---|---|
| $\Gamma^2 = \gamma$ | $f(t) = (C_1 + C_2 t)e^{-\Gamma t}$ |
| $\Gamma^2 < \gamma$ | $f(t) = C_1 e^{-\Gamma t} \cos(\sqrt{\gamma - \Gamma^2} t + C_2)$ |
| $\Gamma^2 > \gamma$ | $f(t) = e^{-\Gamma t}(C_1 e^{\sqrt{\Gamma^2 - \gamma} t} + C_2 e^{-\sqrt{\Gamma^2 - \gamma} t})$ |

## DISCUSSION

As is seen from (4), in all solutions the **E** and **B** fields are mutually orthogonal and spatially inhomogeneous. The **E** field is purely azimuthal, vanishes on the $\zeta$ axis (the axis where $\rho = 0$), and is maximum along the circle $\rho = \pi/2$. Oppositely, the **B** field is purely longitudinal, is maximum along the $\zeta$ axis and vanishes on the circle $\rho = \pi/2$. These expressions for the fields are globally possible whenever the topology of the underlying 3-space is either the simply connected 3-sphere $S^3$ or the multiply connected real projective 3-space $P^3$. No other multiply connected 3-space endowed with the elliptical geometry (e.g. the Poincaré dodecahedron) seems appropriate to globaly accomodate these forms of field.

From Table I we immediately distinguish two *static* solutions: one is the trivial no-field solution $\mathbf{E} = \mathbf{B} = 0$, corresponding to $C_1 = C2 = 0$; the other is a pure magnetostatic field with $\mathbf{E} = 0$ and $B_\zeta = 2C_1 \cos\rho$, and belongs to class $\Gamma^2 > \gamma$ with $C^2 = 0$, $\gamma = 0$, $\kappa = -1$, $\lambda = R/2$.

All *non-static* solutions are *standing* Proca waves. Most have exponential damping with increasing time. Nevertheless, in the class $\Gamma^2 > \gamma$, an exception deserves mentioning: when $\gamma < 0$, that is $\kappa = -1$ and $\lambda < R/2$ in eq.(7), the potential $f(t)$ and the Proca fields show an exponential growth as time increases. Three sets of non-static solutions with the quantity $\Delta = E_\phi^2 + c^2 B_\zeta^2$ independent on the location in three-space were encountered: see Table II. Sets **a** and **b** both have $\Delta \propto \exp(-4ct/R)$ (damping along the time), and both contain $\lambda \to \infty$, $\sigma = 4c/R$ (a Maxwell field) as a special case. The set **c** has $\Delta \propto \exp(+4ct/R)$ (increasing along the time). Sets **b** and **c** both contain the special case $\lambda = R/\sqrt{8}$, $\sigma = 0$ (vanishing conductivity).

**TABLE II**

**Parameters for uniform $\Delta(t)$.**

| | | | | |
|---|---|---|---|---|
| a | $\kappa = +1$ | $\sigma = 4c/R + cR/(2\lambda^2)$ | $\lambda$ free | $\epsilon = -1$ |
| b | $\kappa = -1$ | $\sigma = 4c/R - cR/(2\lambda^2)$ | $\lambda \geq R/\sqrt{8}$ | $\epsilon = -1$ |
| c | $\kappa = -1$ | $\sigma = cR/(2\lambda^2) - 4c/R$ | $\lambda \leq R/\sqrt{8}$ | $\epsilon = +1$ |

A few words seem worthwhile, concerning the physical values of the constants $m_\gamma$, $\lambda$, $\sigma$ and $R$.

First recall that the mass $m_\gamma$ and the range $\lambda$ share the quantum correspondence $m_\gamma c = h/\lambda$, where $h = 6.6 \times 10^{-34}$ J s is Planck's constant. Assuming $\lambda \approx R \approx 10^{10}$ l.y. $\approx 10^{26}$ m, then $m_\gamma \approx 10^{-68}$ kg, which is fifteen orders of magnitude smaller than the upper limit obtained by experimental techniques (Goldhaber & Nieto 1971); this amounts to saying that a Proca field with that value for the range $\lambda$ is presently indiscernible from a Maxwell field. From Table II, and still assuming $\lambda \approx R \approx 10^{26}$ m, one should have $\sigma \approx 10^{-17}$/s for systems with $\mathbf{E}^2 + c^2\,\mathbf{B}^2$ homogeneous over the 3-space; this value for the conductivity coincides in order of magnitude with that of ref. (Kar *et al.* 1993), obtained in a different context. To conclude, if we consider the above values for the various constants in the damping harmonic class $\Gamma^2 < \gamma$ in Table I, then the resulting frequency would be $\delta \approx 10^{-18}$ Hz; fields with such a slow variation would seem static.

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## REFERENCES

AHONEN J & ENQVIST K. 1996. Electrical conductivity in the early universe. *Phys Lett B* **382:** 40-44.

GOLDHABER AS & NIETO MM. 1971. Terrestrial and Extraterrestrial Limits on The Photon Mass. *Rev Mod Phys* **43:** 277-296.

KAR G, SINHA M & ROY S. 1993. Maxwell Equations, Nonzero Photon Mass, and Conformal Metric Fluctuation. *Int J Theor Phys* **32:** 593-607.

VIGIER JP. 1990. Evidence For Nonzero Mass Photons Associated With a Vacuum-Induced Dissipative Red-Shift Mechanism. *IEEE Transactions on Plasma Science* **18:** 64-72.