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MATHEMATICAL SCIENCES

Minimum sample size for estimating the Net Promoter Score under a Bayesian approach

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Abstract: At some moment in our lives, we are probably faced with the following question: "How likely is it that you would recommend [company X] to a friend or colleague?". This question is related to the Net Promoter Score (NPS), a simple measure used by several companies as indicator of customer loyalty. Even though it is a well-known measure in the business world, studies that address the statistical properties or the sample size determination problem related to this measure are still scarce. We adopt a Bayesian approach to provide point and interval estimators for the NPS and discuss the determination of the sample size. Computational tools were implemented to use this methodology in practice. An illustrative example with data from financial services is also presented.

Key words: Average length criterion, customer loyalty, Dirichlet distribution multinomial distribution, sample size.

1 - INTRODUCTION

Reichheld (2003) proposed a statistics called Net Promoter Score (NPS) that may be used by a company as an indicator of customer loyalty. The author applied a questionnaire with some questions related to loyalty to a sample of customers of some industries, and with the purchase history of each customer it was possible to determine which questions had the strongest statistical correlation with repeat purchase or referrals. One of these questions performed better in most industries: "How likely is it that you would recommend [company X] to a friend or colleague?". Reichheld (2003) suggested that the response to the this questions must be on a 0 to 10 rating scale. Then, it is considered "promoters" the customers who respond with 9 or 10, "passives" the customers who respond with 7 or 8, and "detractors" the customers who respond with 0 through 6. The idea is that the more "promoters" company X has, the bigger its growth. An estimate of the NPS is computed as the difference between the proportions (or percentages) of "promoters" and "detractors".

Keiningham et al. (2008) discuss the claims that NPS is the single most reliable indicator of a company's ability to grow, and that it is a superior metric to costumer satisfaction. Markoulidakis et al. (2021) approach the customer experience as a NPS classification problem via machine learning algorithms. Rocks (2016) presents a brief summary of some critiques about the NPS, see references therein. We may also cite Eskildsen & Kristensen (2011) and Kristensen & Eskildsen (2014) for related work.

In the context of statistical modeling, Rocks (2016) focus on estimating intervals for the NPS in a frequentist approach via Wald intervals and Score methods. Also, the author performs a study simulation to assess the coverage probability of the proposed interval estimates, and conclude that variations on the adjusted Wald and an iterative Score method performed better.

In Section 2, we present the frequentist approach to make inference about the NPS. In Section 3, we present the Bayesian approach with the multinomial/Dirichlet model, the methodologies to obtain point and interval estimates for the NPS. Also in Section 3, the problem of the minimum sample size determination is discussed and a study simulation is conducted. In Section 4, we present an illustrative example with data on financial services. We conclude with some remarks in Section 5.

2 - FREQUENTIST APPROACH

Let X_1, X_2 and X_3 the respective numbers of detractors, passives and promoters in a customer sample of size n; θ_1, θ_2 and θ_3 the respective proportions in the customer population. Then, the parameter of interest is $\Delta = \theta_3 - \theta_1$, where $\Delta \in [-1, 1]$, and an estimator for this parameter is given by $\text{NPS} = (X_3 - X_1)/n$. The Wald interval with $100(1-\rho)\%$ confidence level is given by $\text{NPS} \pm z_{\rho/2}\sqrt{\frac{\sigma_{\text{NPS}}}{n}}$, where $z_{\rho/2}$ is the quantile of probability $1-\rho/2$ of the standard normal distribution and $\sigma_{\text{NPS}} = \theta_3 + \theta_1 - (\theta_3 - \theta_1)^2$ (Rocks 2016). In this context, when NPS is used to estimate Δ, the error $|\text{NPS} - \Delta|$ is less than or equal to $z_{\rho/2}\sqrt{\frac{\sigma_{\text{NPS}}}{n}}$ with confidence $100(1-\rho)\%$. Then, we may obtain n so that we are $100(1-\rho)\%$ confident that the error in estimating Δ is less than or equal to a specified bound (ϵ) on the error, *i.e.*,

$$z_{\rho/2}\sqrt{\frac{\sigma_{\text{NPS}}}{n}} \leq \epsilon,$$

which implies that

$$n \geq \sigma_{\text{NPS}} \left(\frac{z_{\rho/2}}{\epsilon}\right)^2. \tag{1}$$

Another interval proposed by Rocks (2016) and based on Goodman (1964) is $\text{NPS} \pm \sqrt{q_\rho \frac{\sigma_{\text{NPS}}}{n}}$, where q_ρ is the quantile of probability $1-\rho$ of the chi-squared distribution with two degrees of freedom. In the same way of the previous interval, we obtain that

$$n \geq \sigma_{\text{NPS}} \frac{q_\rho}{\epsilon^2}. \tag{2}$$

Similar expressions to (1) and (2) for n may be obtained from the other intervals proposed by Rocks (2016). But note that, in order to obtain the minimum n we have to set a value for σ_{NPS}, which depends on unknown quantities (θ_1 and θ_3). One solution is to set σ_{NPS} based on previous studies or consider the maximum value of σ_{NPS}, which is 1 (Rocks 2016). Another problem is that these intervals are based on the respective asymptotic distribution in each case, *i.e.*, as the $n \to \infty$, and the computed minimum n may not provide a good approximation to the asymptotic distribution.

In order to circumvent these problems, we propose a Bayesian model in order to make inference for the NPS and to establish a sample size determination methodology. Even though it is a well-known and widely used measure in the business world, studies that address the statistical properties or the sample size determination problem are still scarce. See Rossi & Allenby (2003) for an exposition of the usefulness of the Bayesian methods in marketing.

3 - BAYESIAN APPROACH

Let $\theta = (\theta_1, \theta_2, \theta_3)$, where θ_1, θ_2 and θ_3 are the proportions of detractors, passives and promoters in the customer population, respectively. Then, the NPS in the respective population is given by $\Delta = \theta_3 - \theta_1$, where $\Delta \in [-1, 1]$, which is the parameter of interest. In a sample of n customers we count the number of customers in each category based on their responses for the aforementioned question. Let $X_n = (X_1, X_2, X_3)$, where X_1, X_2 and X_3 are the numbers of customers categorized as detractors, passives and promoters, respectively, in the customer sample of size n.

3.1 - Multinomial/Dirichlet model

Given θ, we assume a multinomial distribution for the counts X_n, and we denote $X_n|\theta \sim \text{Mult}(n, \theta)$. The respective probability distribution is given by

$$\mathbb{P}\left[X_1 = x_1, X_2 = x_2, X_3 = x_3\right] = \frac{n!}{x_1! x_2! x_3!} \theta_1^{x_1} \theta_2^{x_2} \theta_3^{x_3},$$

where $x_1, x_2, x_3 = 0, 1, \ldots, n$ such that $x_1 + x_2 + x_3 = n$, and $\theta_1 + \theta_2 + \theta_3 = 1$.

The natural (conjugate) choice for the prior distribution of θ is a Dirichlet distribution. We denote $\theta \sim \text{Dir}(\alpha)$ and the respective probability density function is given by

$$\pi(\theta) = \frac{\Gamma(\alpha_1 + \alpha_2 + \alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)} \theta_1^{\alpha_1 - 1} \theta_2^{\alpha_2 - 1} \theta_3^{\alpha_3 - 1},$$

where $\theta_1 + \theta_2 + \theta_3 = 1$, $\Gamma(\cdot)$ is the gamma function and $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ is a vector of positive hyperparameters which must be set by the researcher and must reflect the prior knowledge about θ at the moment. In Figures 1 and 2 we present the ternary plots for the Dirichlet distribution for different values for α. For values of α close to zero the distribution of θ concentrates in the corners of the plot, which implies that the distribution of $\Delta = \theta_3 - \theta_1$ will mostly concentrate around the values -1, 0 and 1. As the values in vector α increase but still smaller than 1, the distribution of θ will mostly concentrates on the edges. If $\alpha_1 = \alpha_2 = \alpha_3 = 1$ the distribution of θ is equivalent to a uniform distribution over the 2-simplex. For values of α_1, α_2 and α_3 greater than 1, the distribution of θ becomes increasingly concentrated in a given region of the plot as the values of α_1, α_2 and α_3 increase, where the determination of the region of concentration depends on the values of α_1, α_2 and α_3.

Let $\alpha_0 = \alpha_1 + \alpha_2 + \alpha_3$, Adcock (1987) proposes a method to determine α_0 before sampling if there are two independent estimates $e_1 = (e_{11}, e_{12}, e_{13})$ and $e_2 = (e_{21}, e_{22}, e_{23})$ for θ, and note that the expected value of $Q = (e_1 - e_2)^\top (e_1 - e_2)$ is given by

$$\mathbb{E}\left[Q\right] = \frac{2\left[1 - \sum_{i=1}^{3} m_i^2\right]}{\alpha_0 + 1},$$

where m_i is estimated by $(e_{1i} + e_{2i})/2$, $i = 1, 2, 3$. For example, suppose that $e_1 = (0.32, 0.26, 0.42)$ and $e_2 = (0.27, 0.32, 0.41)$, taking $D = \sum_{i=1}^{3} (e_{1i} - e_{2i})^2$ as an estimate for $\mathbb{E}\left[Q\right]$ and solving the expected value equation for α_0, we have that $\alpha_0 = \left[2\left(1 - \sum_{i=1}^{3} m_i^2\right)/D\right] - 1 \approx 211$, and multiplying this value by m_i, $i = 1, 2, 3$, we obtain $\alpha \approx (62, 61, 88)$. On the other hand, if we have that $e_1 = (0.32, 0.26, 0.42)$

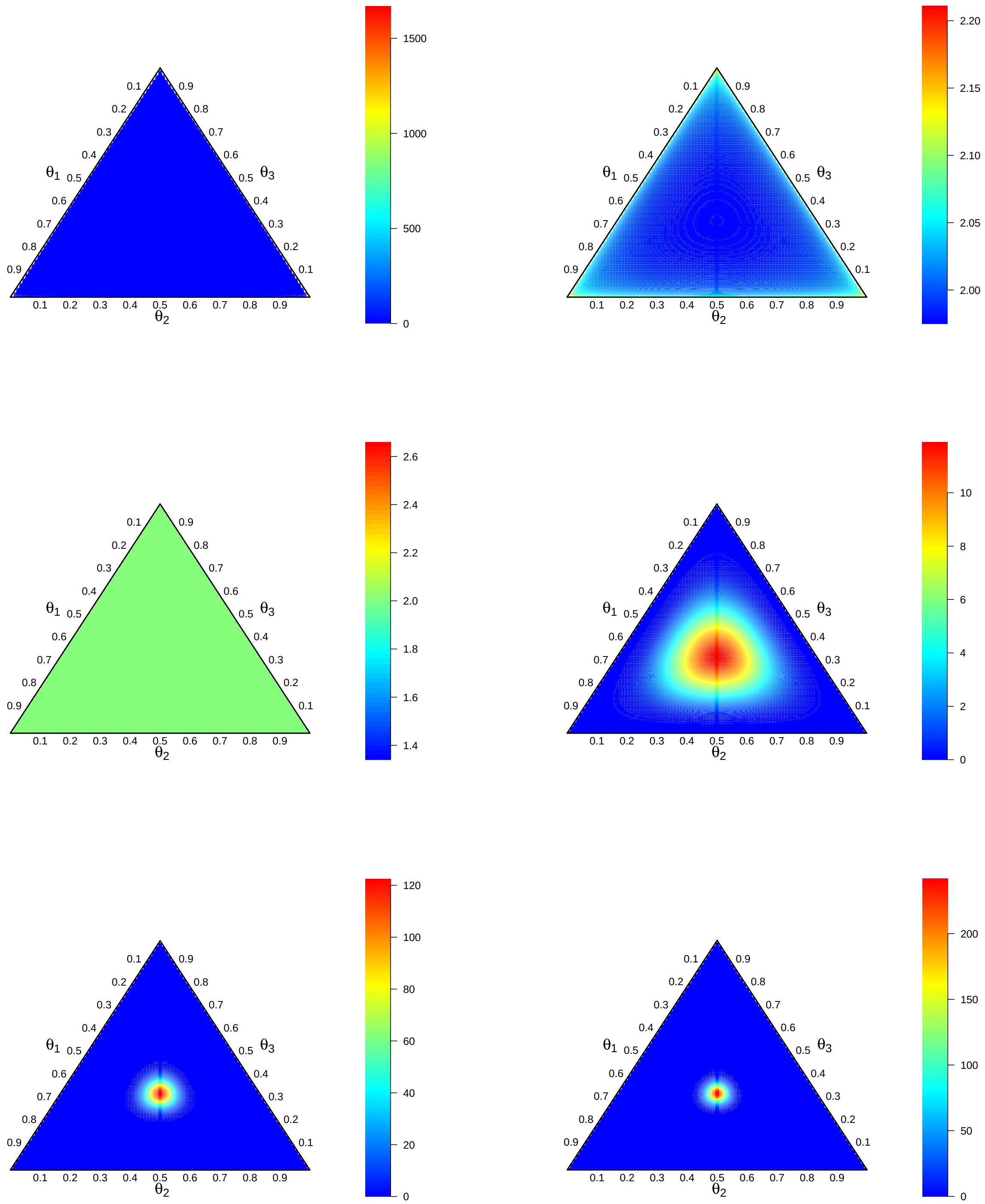

Figure 1. Ternary plots for the Dirichlet distribution with: $\alpha = (0.1, 0.1, 0.1)$ **and** $\alpha = (0.99, 0.99, 0.99)$ **in the top row;** $\alpha = (1, 1, 1)$ **and** $\alpha = (5, 5, 5)$ **in the middle row;** $\alpha = (50, 50, 50)$ **and** $\alpha = (100, 100, 100)$ **in the bottom row.**

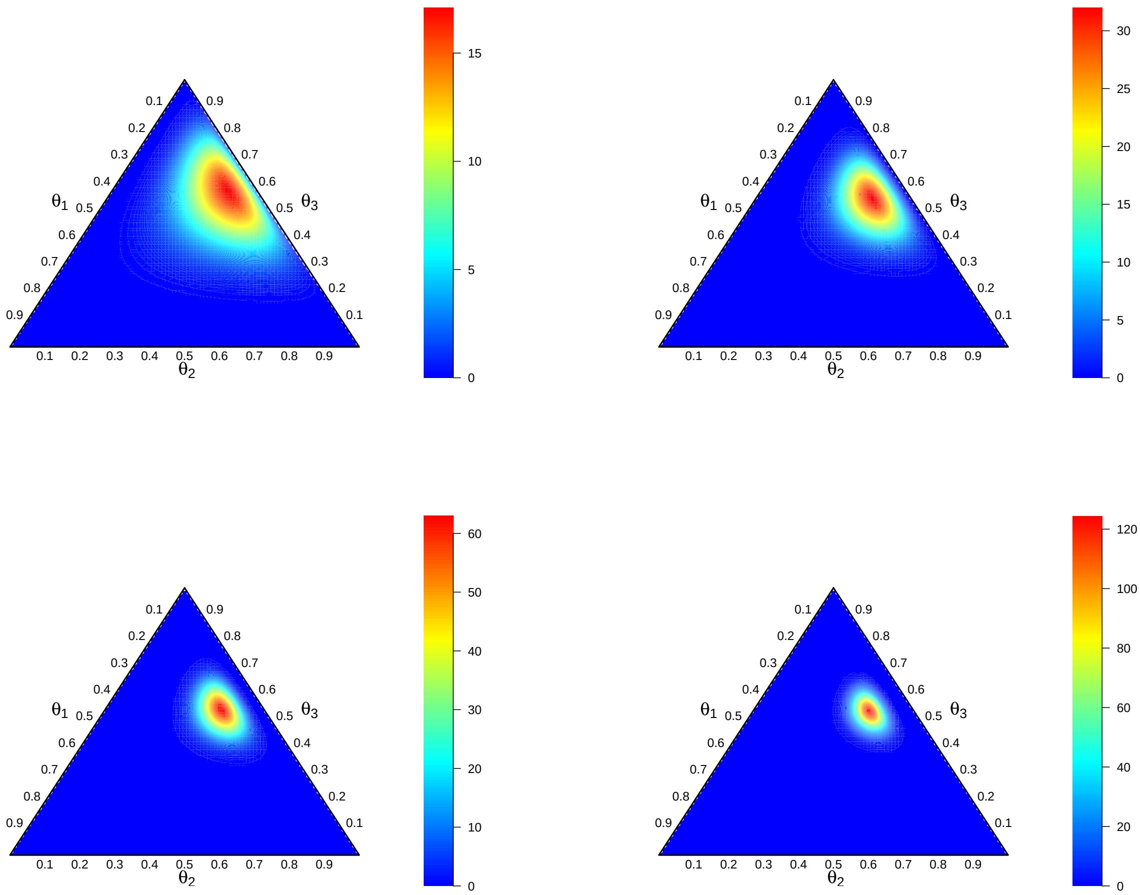

Figure 2. Ternary plots for the Dirichlet distribution with: $\alpha = (2, 5, 8)$ **and** $\alpha = (4, 10, 16)$ **in the top row;** $\alpha = (8, 20, 32)$ **and** $\alpha = (16, 40, 64)$ **in the bottom row.**

and $e_2 = (0.15, 0.52, 0.33)$, we obtain that $\alpha_0 \approx 11$ and $\alpha \approx (3, 4, 4)$, *i.e.*, α_0 increases as the distance between e_1 and e_2 decreases.

Given the multinomial distribution to model the counts and the Dirichlet distribution for θ, the model may be written hierarchically as follows

$$X_n|\theta \sim \text{Mult}(\theta); \quad \theta \sim \text{Dir}(\alpha). \tag{3}$$

In this setting, given a observation x_n of X_n, we have that the posterior distribution for θ is a Dirichlet distribution with parameter $\alpha + x_n$, *i.e.*, $\theta|x_n \sim \text{Dir}(\alpha + x_n)$ (Turkman et al. 2019). Also, Bayesian updating becomes straightforward since the current parameters of the posterior distribution may be used as the hyperparameters of the prior distribution in the next sampling of X_n. Given a way to generate random values from the Dirichlet distribution, this provide us a simple way to draw values from the posterior distribution of Δ in order to obtain, approximately, posterior summaries as the mean, median, variance, quantiles, etc, and make inferences about the NPS.

An algorithm to obtain a sample of size N from the posterior distribution of the Δ is outlined as follows.

1. Set the values of α, x_n and N (*e.g.*, $N = 1000$).
2. Draw a value of $\theta = (\theta_1, \theta_2, \theta_3)$ from the Dirichlet distribution with parameter $\alpha + x_n$.
3. Compute $\Delta = \theta_3 - \theta_1$ and keep this value.
4. Repeat Steps 2-3 N times.

It is well known that the marginal distributions of a Dirichlet distribution are beta distributions (Kotz et al. 2000). Let $\alpha^* = \alpha + x_n = (\alpha_1^*, \alpha_2^*, \alpha_3^*)^\top$. Then, it follows that

$$\theta_1|x_n \sim \text{Beta}(\alpha_1^*, \alpha_2^* + \alpha_3^*) \quad \text{and} \quad \theta_3|x_n \sim \text{Beta}(\alpha_3^*, \alpha_1^* + \alpha_2^*),$$

which give us that the mean of the posterior distribution of the NPS is

$$\mathbb{E}\left[\Delta|x_n\right] = \mathbb{E}\left[\theta_3 - \theta_1|x_n\right] = \frac{\alpha_3^* - \alpha_1^*}{\alpha_0^*}, \tag{4}$$

where $\alpha_0^* = \alpha_1^* + \alpha_2^* + \alpha_3^*$. This mean may be used as a point estimator for the NPS. The respective variance is given by

$$\begin{aligned} \text{Var}\left[\Delta|x_n\right] &= \text{Var}\left[\theta_3|x_n\right] + \text{Var}\left[\theta_1|x_n\right] - 2\text{Cov}(\theta_3, \theta_1|x_n) \\ &= \frac{\alpha_1^*\alpha_2^* + \alpha_2^*\alpha_3^* + 4\alpha_1^*\alpha_3^*}{(\alpha_0^*)^2(\alpha_0^* + 1)}. \end{aligned} \tag{5}$$

A credible interval that we may construct is based on (4) and (5), *i.e.*, $\mathbb{E}\left[\Delta|x_n\right] \pm \gamma\sqrt{\text{Var}\left[\Delta|x_n\right]}$, where $\gamma > 0$ is a fixed constant that controls the number of posterior standard deviations within the credible interval. This type of credible interval may be derived from decision theory, where a loss function is composed of measures of bias and discrepancy, and γ is related to the calibration of the trade-off between these measures (Rice et al. 2008). Also in the context of decision theory, the γ may be determined for a fixed coverage probability using asymptotic properties for posterior distributions. For example, see Costa et al. (2021b, Section 5) for more details. We developed an Excel spreadsheet that computes this credible interval and a point estimate based on (4) and (5). See Supplementary Material for more details.

Another credible interval may be specified by the highest posterior density (HPD) interval. In this case we use a Monte Carlo approach to approximate the HPD interval. In other words, we use a sample drawn from the posterior distribution of Δ, which may be easily done since the posterior distribution is a Dirichlet distribution. See Turkman et al. (2019, pgs. 47-48) for more details.

3.2 - Minimum sample size

To determine the minimum sample size required to estimate Δ with a pre-specified precision, we consider a criterion based on the average length of credible intervals. The posterior credible interval accounts for the magnitude of the NPS and this may help the company to know when to perform a gap analysis and create a business action plan in order to improve the NPS, *i.e.*, increase the NPS until the company has more promoters than detractors ($\Delta > 0$).

Let $a(x_n)$ and $b(x_n)$ be the lower and upper bounds of the HPD interval for Δ. The rationale here is to set the minimum Bayesian coverage probability $1-\rho$ and obtain the minimum sample size by requiring that the length of the HPD interval $\ell(x_n) = b(x_n) - a(x_n)$ be such that

$$\mathbb{E}\left[\ell(X_n)\right] \leq \ell_{\max},$$

where $\ell_{\max}$ is the maximum admissible length for the HPD interval and the expected value is computed based on the marginal probability function of the outcomes (X_n). This is called average length criterion (ALC). See Costa et al. (2021a) and references therein for more details about this criterion.

Since it is impractical to obtain analytically the lower and upper bounds of the HPD interval for Δ, we use a Monte Carlo approach (Chen & Shao 1999) to obtain the respective bounds as well as the the respective expected value.

An algorithm to obtain the minimum sample size satisfying this criterion is outlined as follows.

1. Set values for $\ell_{\max}$, α, ρ and take $n = 1$.
2. Draw a sample of size L (*e.g.*, $L = 1000$) of x_n; to draw x_n, first draw one value of θ from the Dirichlet distribution with parameter α and given this value, draw x_n from the multinomial distribution with parameter θ.
3. Obtain the HPD interval of probability $1-\rho$ for each x_n and then the respective interval length: for each vector drawn in Step 2, obtain the lower and upper bounds of the HPD interval of probability $1-\rho$ as indicated in Chen & Shao (1999). Then, compute the difference between the upper and lower bounds for each vector drawn, in order to obtain the interval lengths.
4. Compute the average of the L HPD interval lengths.
5. If this average is lower or equal to $\ell_{\max}$, stop. The value n obtained in this step is the required value. Otherwise, set $n = n + 1$ and return to Step 2.

We developed an R package (R Core Team 2022) which provides a function to obtain point and interval estimates via Monte Carlo simulation as discussed in the above section. Also, the package have a function to compute the minimum sample size to estimate the NPS through HPD interval via ALC (see Supplementary Material).

In Tables I and II, we present the minimum sample size to estimate the NPS using the HPD interval computed via ALC for all the scenarios for the prior distribution of θ presented in Figures 1 and 2, and some values of $\ell_{\max}$ and ρ. In the algorithm to obtain the minimum n, we consider $L = 1000$ and in the Step 3 we draw samples of size $N = 100$ from the posterior distribution of the NPS in order to obtain the HPD interval bounds. For fixed ρ ($\ell_{\max}$), the minimum n decreases as $\ell_{\max}$ (ρ) increases, as expected (Tables I and II).

Also in Tables I and II, we observe that the minimum n increases when α changes from $(0.1, 0.1, 0.1)$ to $(0.99, 0.99, 0.99)$, which makes sense since the first case represents a Dirichlet distribution concentrated in the corners that implies a prior distribution for Δ mostly concentrated around the values -1, 0, and 1; and the second case represents a Dirichlet distribution mostly concentrated on the edges. For $\alpha_i \geq 1$, $i = 1, 2, 3$, we observe that the minimum n increases as $\alpha_0 = \sum_{i=1}^{3} \alpha_i$ increases until some point and after that point the minimum n decreases. It seems that there is some value k_0 in which the minimum n increases as α_0 approaches k_0 from left, and for $\alpha_0 > k_0$ the minimum n decreases as α_0 approaches infinity. Also, we observe that the magnitude of k_0 depends on $\ell_{\max}$, the smaller the $\ell_{\max}$, the larger the k_0, *i.e.*, when $\alpha_0 < k_0$ the prior knowledge is not enough to make the minimum n start decreasing and satisfy the ALC for the fixed values of $\ell_{\max}$ and ρ. This is consistent with the interpretation that α_0 is a "prior sample size" and that it may be viewed as a measure of the quality of the prior knowledge (Adcock 1987).

For the adopted model parameters, the running time to compute the minimum n varied from 47 seconds to 5.52 hours, depending on the setting. The smaller the values of $\ell_{\max}$ and/or ρ, the larger the running time. The computers that have been used have the following characteristics: (i) OS Linux Ubuntu 20.04, RAM 7.7 GB, processor AMD PRO A8-8600B; and (ii) OS Linux Ubuntu 22.04, RAM 5.6 GB, processor AMD Ryzen 5 5500U.

3.3 - Simulation study

We conduct a simulation study to verify whether the HPD intervals obtained with the sample sizes proposed in Section 3.2 satisfy the ALC.

For each sample size obtained via the ALC displayed in Tables I and II, and the respective values of α, ρ and $\ell_{\max}$, we perform the following steps: (i) we draw θ from the Dirichlet distribution and compute the correspondent NPS $\Delta = \theta_3 - \theta_1$; (ii) given the θ, we draw x_n from the multinomial distribution; (iii) given α and x_n, we draw a sample of size 1000 from the posterior distribution of Δ and obtain the respective HPD interval with probability $1-\rho$; (iv) we compute the length of the respective HPD interval and verify if Δ belongs to this interval; (v) we repeat the steps (i)-(iv) 1000 times, and we compute the average of the lengths of the HPD intervals and the proportion of times that Δ belonged to the HPD interval.

In Tables III-VI we displayed the simulation results. As expected, the ALC based lengths and coverage probabilities of HPD intervals estimated via the simulation study are close to the respective values of $\ell_{\max}$ and $1 - \rho$ for each sample size in Tables I and II. For the estimated ALC based lengths of HPD intervals, in general the values obtained in the simulation study are slightly larger than the respective $\ell_{\max}$, with a difference in the third decimal place. On the other hand, for the estimated ALC based coverage probabilities, in general the values obtained are slightly smaller than the respective $1 - \rho$.

4 - ILLUSTRATIVE EXAMPLE

In the situation where the sample is not obtained yet, we may determine the sample size to obtain a HPD interval for the NPS via ALC by setting α_1, α_2, α_3, $\ell_{\max}$ and ρ. For example, if we consider $\alpha_1 = \alpha_2 = \alpha_3 = 1$, $\ell_{\max} = 0.10$ and $\rho = 0.05$, the minimum sample size is 655 (Table I), *i.e.*, to obtain a HPD interval with maximum length equals to 0.10 and respective coverage probability equals to 0.95, we should ask 655 people the NPS question and then categorize them into detractors, passives and promoters in order to obtain the observed values of X_1, X_2 and X_3, respectively.

Table I. **ALC based minimum sample size to estimate the NPS through the HPD interval via multinomial/Dirichlet model with $\alpha = (\alpha_1, \alpha_2, \alpha_3)$.**

	$(0.1, 0.1, 0.1)$				$(0.99, 0.99, 0.99)$		
$\ell_{\max} \setminus \rho$	0.01	0.05	0.10		0.01	0.05	0.10
0.02	4406	2482	1753		28406	16048	11222
0.04	1089	629	415		7073	4033	2836
0.06	531	295	200		3148	1801	1270
0.08	311	167	109		1822	995	690
0.10	202	108	70		1148	639	457
0.12	147	78	48		808	458	319
0.14	110	57	36		593	333	235
0.16	90	43	27		447	258	178
0.18	71	32	23		355	201	141
0.20	57	27	18		290	161	115
	$(1, 1, 1)$				$(5, 5, 5)$		
$\ell_{\max} \setminus \rho$	0.01	0.05	0.10		0.01	0.05	0.10
0.02	28199	16034	11328		37737	21516	15157
0.04	7153	4085	2805		9521	5396	3810
0.06	3222	1808	1272		4243	2386	1681
0.08	1786	992	721		2391	1330	944
0.10	1158	655	460		1515	860	599
0.12	807	460	315		1046	590	409
0.14	591	334	234		764	433	298
0.16	452	249	179		585	326	224
0.18	359	203	142		464	254	175
0.20	291	164	114		371	204	139
	$(50, 50, 50)$				$(100, 100, 100)$		
$\ell_{\max} \setminus \rho$	0.01	0.05	0.10		0.01	0.05	0.10
0.02	40494	22924	16116		40670	22987	16007
0.04	10085	5666	3951		10004	5532	3796
0.06	4411	2435	1668		4286	2285	1520
0.08	2422	1313	876		2283	1157	728
0.10	1505	781	501		1355	637	358
0.12	991	500	310		848	348	159
0.14	685	326	187		550	178	37
0.16	490	216	107		346	67	2
0.18	357	138	53		208	2	2
0.20	266	82	15		116	2	2

Table II. **ALC based minimum sample size to estimate the NPS through the HPD interval via multinomial/Dirichlet model with $\alpha = (\alpha_1, \alpha_2, \alpha_3)$.**

	$(2, 5, 8)$				$(4, 10, 16)$		
$\ell_{\max} \setminus \rho$	0.01	0.05	0.10		0.01	0.05	0.10
0.02	28494	16105	11346		29723	16796	11841
0.04	7116	4018	2816		7509	4207	2949
0.06	3198	1790	1258		3303	1857	1304
0.08	1779	1004	698		1847	1033	720
0.10	1137	636	442		1169	645	449
0.12	788	442	307		809	449	305
0.14	577	322	225		586	322	217
0.16	433	243	166		444	236	156
0.18	342	189	130		340	182	119
0.20	275	150	100		275	139	91
	$(8, 20, 32)$				$(16, 40, 64)$		
$\ell_{\max} \setminus \rho$	0.01	0.05	0.10		0.01	0.05	0.10
0.02	30537	17235	12072		30811	17349	12124
0.04	7614	4256	2988		7640	4291	2985
0.06	3368	1874	1302		3343	1842	1261
0.08	1840	1034	708		1833	982	654
0.10	1180	643	431		1127	587	373
0.12	801	425	284		742	373	223
0.14	574	297	190		517	242	136
0.16	426	215	133		372	157	76
0.18	323	156	93		264	101	35
0.20	248	116	63		194	56	7

Table III. ALC based length of HPD intervals estimated via simulation for some scenarios under the multinomial/Dirichlet model using sample sizes displayed in Table I.

	$(0.1, 0.1, 0.1)$			$(0.99, 0.99, 0.99)$		
$\ell_{max} \setminus \rho$	0.01	0.05	0.10	0.01	0.05	0.10
0.02	0.0219	0.0216	0.0227	0.0203	0.0209	0.0210
0.04	0.0429	0.0440	0.0440	0.0407	0.0417	0.0420
0.06	0.0604	0.0660	0.0627	0.0612	0.0623	0.0632
0.08	0.0821	0.0837	0.0828	0.0815	0.0846	0.0848
0.10	0.1016	0.0986	0.1041	0.1005	0.1060	0.1040
0.12	0.1217	0.1235	0.1202	0.1218	0.1239	0.1262
0.14	0.1442	0.1391	0.1445	0.1417	0.1446	0.1441
0.16	0.1542	0.1619	0.1677	0.1638	0.1650	0.1661
0.18	0.1817	0.1861	0.1782	0.1825	0.1857	0.1845
0.20	0.2034	0.2068	0.1959	0.2013	0.2093	0.2055
	$(1, 1, 1)$			$(5, 5, 5)$		
$\ell_{max} \setminus \rho$	0.01	0.05	0.10	0.01	0.05	0.10
0.02	0.0207	0.0212	0.0208	0.0206	0.0207	0.0208
0.04	0.0407	0.0417	0.0426	0.0408	0.0415	0.0412
0.06	0.0612	0.0626	0.0628	0.0608	0.0622	0.0625
0.08	0.0840	0.0856	0.0819	0.0811	0.0829	0.0826
0.10	0.1027	0.1034	0.1044	0.1022	0.1026	0.1035
0.12	0.1230	0.1238	0.1231	0.1225	0.1237	0.1243
0.14	0.1425	0.1463	0.1450	0.1432	0.1438	0.1445
0.16	0.1630	0.1672	0.1665	0.1616	0.1647	0.1659
0.18	0.1818	0.1834	0.1837	0.1821	0.1869	0.1853
0.20	0.2044	0.2056	0.2075	0.2020	0.2051	0.2061
	$(50, 50, 50)$			$(100, 100, 100)$		
$\ell_{max} \setminus \rho$	0.01	0.05	0.10	0.01	0.05	0.10
0.02	0.0204	0.0207	0.0207	0.0203	0.0207	0.0207
0.04	0.0408	0.0414	0.0412	0.0407	0.0413	0.0414
0.06	0.0609	0.0618	0.0620	0.0609	0.0620	0.0622
0.08	0.0813	0.0824	0.0826	0.0811	0.0827	0.0827
0.10	0.1011	0.1031	0.1037	0.1013	0.1030	0.1033
0.12	0.1220	0.1234	0.1234	0.1219	0.1238	0.1235
0.14	0.1426	0.1444	0.1441	0.1415	0.1439	0.1446
0.16	0.1626	0.1644	0.1648	0.1620	0.1645	0.1527
0.18	0.1826	0.1853	0.1855	0.1824	0.1815	0.1525
0.20	0.2017	0.2064	0.2056	0.2022	0.1812	0.1523

Table IV. ALC based coverage probability of HPD intervals estimated via simulation for some scenarios under the multinomial/Dirichlet model using sample sizes displayed in Table I.

	$(0.1, 0.1, 0.1)$			$(0.99, 0.99, 0.99)$		
$\ell_{max} \setminus \rho$	0.01	0.05	0.10	0.01	0.05	0.10
0.02	0.982	0.952	0.892	0.992	0.941	0.894
0.04	0.993	0.950	0.900	0.987	0.941	0.904
0.06	0.989	0.946	0.899	0.981	0.942	0.901
0.08	0.977	0.952	0.901	0.990	0.951	0.885
0.10	0.990	0.944	0.887	0.982	0.945	0.901
0.12	0.989	0.953	0.906	0.985	0.932	0.876
0.14	0.987	0.951	0.896	0.991	0.948	0.885
0.16	0.991	0.939	0.882	0.984	0.950	0.883
0.18	0.986	0.941	0.882	0.987	0.944	0.907
0.20	0.987	0.952	0.900	0.996	0.945	0.890
	$(1, 1, 1)$			$(5, 5, 5)$		
$\ell_{max} \setminus \rho$	0.01	0.05	0.10	0.01	0.05	0.10
0.02	0.992	0.942	0.889	0.982	0.939	0.889
0.04	0.989	0.942	0.889	0.996	0.943	0.882
0.06	0.986	0.944	0.901	0.987	0.944	0.894
0.08	0.987	0.942	0.893	0.994	0.944	0.886
0.10	0.989	0.955	0.898	0.988	0.948	0.904
0.12	0.984	0.952	0.883	0.987	0.955	0.903
0.14	0.988	0.950	0.878	0.992	0.944	0.908
0.16	0.988	0.941	0.895	0.985	0.931	0.899
0.18	0.980	0.949	0.908	0.990	0.943	0.890
0.20	0.984	0.955	0.900	0.985	0.949	0.878
	$(50, 50, 50)$			$(100, 100, 100)$		
$\ell_{max} \setminus \rho$	0.01	0.05	0.10	0.01	0.05	0.10
0.02	0.985	0.937	0.906	0.985	0.942	0.897
0.04	0.986	0.935	0.868	0.986	0.939	0.894
0.06	0.994	0.953	0.883	0.991	0.933	0.910
0.08	0.991	0.950	0.898	0.990	0.945	0.892
0.10	0.990	0.939	0.911	0.991	0.931	0.891
0.12	0.988	0.944	0.888	0.983	0.943	0.893
0.14	0.988	0.947	0.894	0.988	0.932	0.889
0.16	0.984	0.952	0.905	0.983	0.950	0.891
0.18	0.990	0.942	0.901	0.985	0.963	0.898
0.20	0.993	0.944	0.899	0.986	0.949	0.875

Table V. ALC based length of HPD intervals estimated via simulation for some scenarios under the multinomial/Dirichlet model using sample sizes displayed in Table II.

	$(2,5,8)$			$(4,10,16)$		
$\ell_{\max} \setminus \rho$	0.01	0.05	0.10	0.01	0.05	0.10
0.02	0.0205	0.0209	0.0207	0.0205	0.0208	0.0207
0.04	0.0409	0.0417	0.0414	0.0406	0.0415	0.0415
0.06	0.0604	0.0625	0.0620	0.0608	0.0621	0.0620
0.08	0.0818	0.0834	0.0823	0.0816	0.0826	0.0830
0.10	0.1022	0.1038	0.1025	0.1021	0.1038	0.1029
0.12	0.1220	0.1236	0.1232	0.1218	0.1239	0.1233
0.14	0.1415	0.1438	0.1433	0.1419	0.1438	0.1442
0.16	0.1623	0.1632	0.1664	0.1616	0.1650	0.1661
0.18	0.1825	0.1845	0.1833	0.1840	0.1843	0.1852
0.20	0.2003	0.2040	0.2061	0.2024	0.2083	0.2055
	$(8,20,32)$			$(16,40,64)$		
$\ell_{\max} \setminus \rho$	0.01	0.05	0.10	0.01	0.05	0.10
0.02	0.0203	0.0207	0.0208	0.0203	0.0207	0.0208
0.04	0.0408	0.0417	0.0414	0.0405	0.0413	0.0412
0.06	0.0608	0.0620	0.0623	0.0610	0.0617	0.0620
0.08	0.0821	0.0822	0.0829	0.0813	0.0826	0.0828
0.10	0.1013	0.1029	0.1036	0.1012	0.1032	0.1035
0.12	0.1217	0.1238	0.1232	0.1222	0.1234	0.1244
0.14	0.1414	0.1447	0.1443	0.1419	0.1438	0.1439
0.16	0.1616	0.1644	0.1646	0.1610	0.1645	0.1645
0.18	0.1815	0.1852	0.1844	0.1819	0.1840	0.1847
0.20	0.2028	0.2051	0.2061	0.2019	0.2065	0.2040

Table VI. ALC based coverage probability of HPD intervals estimated via simulation for some scenarios under the multinomial/Dirichlet model using sample sizes displayed in Table II.

	$(2,5,8)$			$(4,10,16)$		
$\ell_{\max} \setminus \rho$	0.01	0.05	0.10	0.01	0.05	0.10
0.02	0.990	0.947	0.898	0.981	0.946	0.889
0.04	0.987	0.954	0.891	0.987	0.934	0.893
0.06	0.985	0.930	0.906	0.986	0.946	0.898
0.08	0.985	0.941	0.907	0.988	0.941	0.884
0.10	0.987	0.948	0.899	0.986	0.949	0.891
0.12	0.990	0.939	0.892	0.986	0.947	0.881
0.14	0.981	0.948	0.909	0.989	0.942	0.887
0.16	0.981	0.934	0.897	0.991	0.945	0.897
0.18	0.986	0.956	0.892	0.980	0.938	0.894
0.20	0.988	0.955	0.895	0.993	0.943	0.896
	$(8,20,32)$			$(16,40,64)$		
$\ell_{\max} \setminus \rho$	0.01	0.05	0.10	0.01	0.05	0.10
0.02	0.990	0.944	0.885	0.989	0.944	0.902
0.04	0.988	0.939	0.903	0.986	0.940	0.881
0.06	0.993	0.944	0.898	0.994	0.939	0.891
0.08	0.984	0.947	0.894	0.985	0.941	0.883
0.10	0.983	0.943	0.890	0.989	0.930	0.874
0.12	0.988	0.954	0.907	0.990	0.950	0.893
0.14	0.989	0.951	0.893	0.987	0.938	0.879
0.16	0.993	0.956	0.899	0.987	0.954	0.887
0.18	0.990	0.945	0.899	0.984	0.956	0.890
0.20	0.985	0.950	0.887	0.984	0.940	0.895

Given the difficult to obtain a real NPS dataset from a company because such a information is very sensitive, we consider a hypothetical dataset on financial services in three markets in the year of 2021 (see Supplementary Material) to mimic the application of the methods in a real situation.

To illustrate the methodology and the Bayesian updating, we consider the data from the first and second quarter of the Mexico market. For the first quarter we have no prior knowledge, then we set $\alpha_1 = \alpha_2 = \alpha_3 = 1$. For this quarter the numbers of detractors, passives and promoters are 136, 82 and 188, respectively, which implies a posterior Dirichlet distribution with vector parameter $\alpha^* = (137, 83, 189)^\top$ for θ. Drawing a sample of size $N = 1000$ from the respective posterior distribution of the NPS and computing its summaries, we have that a point estimate for the NPS is 0.127 and the HPD 95% interval is [0.038, 0.206]. For the second quarter, we may use the posterior parameter of the first quarter as the prior parameter for the current quarter, *i.e.*, $\alpha_1 = 137$, $\alpha_2 = 83$ and $\alpha_3 = 189$. For the second quarter the numbers of detractors, passives and promoters are 133, 96 and 190, respectively, which implies a posterior Dirichlet distribution with vector parameter $\alpha^* = (270, 179, 379)^\top$ for θ. Again, drawing a sample of size $N = 1000$ for the respective posterior distribution of the NPS, we have a point estimate of 0.131 for the NPS and the HPD 95% interval is [0.072, 0.192]. All these results were obtained via the developed R package. Another way to obtain point and interval estimates for the NPS without needing simulation methods is to compute (4) and (5) for this data, as discussed in Section 3.1.

5 - CONCLUDING REMARKS

We discussed the sample size determination for estimating the NPS. To approach this problem we consider a Bayesian approach via a multinomial/Dirichlet model and the average length criterion. We provide point and interval estimators for the NPS as in closed forms or via drawing a sample from the posterior distribution of the NPS and computing its summaries. A simulation study is conducted to verify whether the HPD intervals obtained with the proposed minimum sample sizes satisfy the ALC. Also, the Bayesian approach makes the inference updating becomes straightforward as illustrated in Section 4, *i.e.*, a sequential procedure to estimate the NPS. Computational tools were developed to use these methodologies in practice.

6 - SUPPLEMENTARY MATERIAL

The Excel spreadsheet is available at https://doi.org/10.5281/zenodo.7679211. The R package is available at https://github.com/eliardocosta/BayesNPS (DOI: 10.5281/zenodo.7617770). The data used in the illustrative example is available at https://www.kaggle.com/code/charlottetu/net-promoter-score/.

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