



Structural Properties of the Brazilian Air Transportation Network

GUILHERME S. COUTO¹, ANA PAULA COUTO DA SILVA²,
LINNYER B. RUIZ¹ and FABRÍCIO BENEVENUTO²

¹Universidade Estadual de Maringá, Centro de Tecnologia, Departamento de Informática,
Avenida Colombo, 5790, 87020-900 Maringá, PR, Brasil

²Departamento de Ciência da Computação, Universidade Federal de Minas Gerais,
Avenida Antônio Carlos, 6627, Pampulha, 31270-010 Belo Horizonte, MG, Brasil

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ABSTRACT

The air transportation network in a country has a great impact on the local, national and global economy. In this paper, we analyze the air transportation network in Brazil with complex network features to better understand its characteristics. In our analysis, we built networks composed either by national or by international flights. We also consider the network when both types of flights are put together. Interesting conclusions emerge from our analysis. For instance, Viracopos Airport (Campinas City) is the most central and connected airport on the national flights network. Any operational problem in this airport separates the Brazilian national network into six distinct subnetworks. Moreover, the Brazilian air transportation network exhibits small world characteristics and national connections network follows a power law distribution. Therefore, our analysis sheds light on the current Brazilian air transportation infrastructure, bringing a novel understanding that may help face the recent fast growth in the usage of the Brazilian transport network.

Key words: Brazilian air transportation network, network science, structural network analysis, complex networks.

INTRODUCTION

Transport networks play an important role on the daily lives of populations. In addition to the impact on the country's economy as well as on its development, transport networks improve people's quality of life. Among all transport networks, air transportation network is one of the most important, connecting people in a fast and safe way. In some countries, such as the USA, the use of airplanes to travel from place to place comes from many decades ago. However,

some countries experimented an explosion on using air transportation system only few years ago (ANAC 2013).

Brazil is an example of a country that has had an impressive growth in the usage of the air transportation network in the last 10 years. This growth is due to two main factors: the improvement of Brazilian family income and the decrease of the price of airline tickets. In 2003, 37.2 million passengers flew (29 million and 8 million passengers for national and international flights, respectively). In 2012, the number of passengers increased to 107 million (\approx 89 million and 18

Correspondence to: Ana Paula Couto da Silva
E-mail: ana.coutosilva@dcc.ufmg.br

million passengers for national and international flights, respectively). Likewise, the total number of flights almost doubled from 611,091 flights in 2003 (534 thousand national flights and 76 thousand international flights) to 1,126,907 flights in 2012 (989 thousand national flights and 137 thousand international flights) (ANAC 2013).

As the most of air transportation networks, the Brazilian network has a complex structure, with dozens of airports and airline companies, operating thousand of national and international flights. Understanding the underlying structural properties of this network is crucial to properly face its fast growth in the last years. Some of the challenges to be faced are, for instance, modernize the airports' infrastructure that are outdated and establish better flight routes in order to improve the quality (Pacheco and Fernandes 2003).

Even though there are many governmental analysis on this topic, we believe that an analysis of the Brazilian air transportation structure, at a network level, is still lacking. The overall analysis of the network enables the identification of the most important and central airports, as well as, the infrastructure robustness under airports fail due to, for instance, weather conditions. Furthermore, it is not clear how the properties of the Brazilian air transportation network compare with other country networks. A complex network analysis (Newman 2003, Reka and Barabási 2002) provides an ideal framework to pursue such a study. To the best of our knowledge, this is the first study that provides a deeper analysis of the Brazilian air transportation using a large number of complex networks features.

For that purpose, we have collected data from *Agência Nacional de Aviação Civil* (ANAC)¹, the civil aviation authority that is responsible for regulating the air transport in Brazil. We built three networks of nodes (representing the airports) and established links between pairs of airports

connected by flights of 51 airline companies that operate in the Brazilian airspace (including the four largest Brazilian airline companies: TAM, Gol/Webjet, Azul/Trip, Avianca). We hope that our analysis sheds light on the current Brazilian air transportation infrastructure, bringing a novel perspective to understanding its main properties and characteristics. Our main findings are:

- The Brazilian Air Transportation Network exhibits small world characteristics with low average shortest path length and high clustering coefficient;
- Airport connections follow a power law distribution, with few hubs connected to many low-degree neighbors;
- Viracopos Airport (Campinas City) is the most connected and central airport in the national flights network, being part of a large number of shortest routes;
- Brazilian travelers need, on average, 3 connection flights to reach their destinations. The maximum number of connections is 7, from Confresa Airport (state of Mato Grosso) to Pato de Minas Airport (state of Minas Gerais);
- The most central airports are concentrated on the Southeast and on the Brazilian coastal region. To reach cities not in these regions many hops are mandatory;
- Viracopos Airport outage breaks the network into 6 subnetworks, affecting 10% of the passengers.

The rest of the paper is organized as follows. In Section 2 (See Related Work), we review the main results found on air transportation networks characterization. Afterward, we describe the data we collected and how we modeled the air transportation network in Section 3 (See The Air Transportation Network). In Section 4 (See Air Transportation Network Analysis and Discussion) we discuss our results and their implications. Lastly, in Section 5 (See Conclusions), we present our main conclusions and directions for future work.

¹ www.anac.gov.br

RELATED WORK

Several papers in the literature have devoted their attention to characterizing and analyzing the airline network from a national and worldwide point of view. In (Guimer et al. 2005) authors analyzed the global structure of the worldwide air transportation network. They found that the worldwide air transportation network follows a scale-free and small-world characteristics and the most connected cities are not necessarily the most central ones. Furthermore, authors identified each global role of a city based on its pattern of intercommunity and intracommunity connections. This result enabled the creation of a scale-specific representation of the whole network.

The main purpose of (Cheung and Gunes 2012) was to analyze the social network features of the U.S. air transportation network. Authors showed that the network exhibits small-world characteristics and, on average, travelers experienced 2 transfers before reaching their destinations. Over the past two decades, network has grown through the years as the number of airports and the number of flight routes between airports has increased to meet customer demands. They also show that the air transportation network, unlike other examples of networks, has only a partial power law degree distribution.

Authors in (Bagler 2008), instead, analyzed the air transportation network in India. This network also has a small-world characteristic, with some airports acting as hubs connected to low-degree neighbors. Chinese airport network is analyzed in (Li and Cai 2004) and it also follows a small-world model. Interestingly, the cumulative degree distributions of both directed and undirected networks obey two-regime power laws with different exponents.

The Italian Airport Network was analyzed in (Guida and Maria 2007). The topological properties of the resulting network have been examined leading to the confirmation of a scale-

free behavior in the connection distribution. However, the scale-free behavior turned out to be a little bit different from the ones already reported, suggesting that the growth mechanism model underlying the network could be different from the ones proposed so far. This consideration is due to the fact that the outcomes of the investigation strongly suggest a fractal structure for this network. Moreover, the paper shows that the clustering coefficient is rather comparable or a little bit smaller than those for a random network, differently from what occurs to some other, where the clustering coefficients are larger than the corresponding random values.

Some works that analyze the Brazilian air transportation network can be found in (Pacheco and Fernandes 2003, Costa et al. 2010, Oliveira et al. 2013). Authors in (Pacheco and Fernandes 2003) analyzed how to improve the infrastructure of Brazilian airports. Authors in (Costa et al. 2010) pointed out how many hubs the Brazilian air transportation network should have to improve its infrastructure quality. Furthermore, authors also indicated the main airports which should be transformed into hubs.

The results more in line with our work are presented in (Oliveira et al. 2013). In that work, authors analyze the Brazilian air transportation system. However, differently of our focus on analyzing the global and local characteristic of the network topology, authors focus on studying the hub organization of airports in Brazil. They show that Guarulhos Airport in São Paulo (GRU) has a crucial role in terms of number of flights and connections. Then, they investigated the robustness of the network with a single hub, by analyzing the impact of the removal of GRU from the network.

Overall, to the best of our knowledge, a network analysis about the Brazilian airport system is missing in the literature. Thus, our work is complementary to the (Pacheco and Fernandes 2003, Costa et al. 2010, Oliveira et al. 2013) efforts.

THE AIR TRANSPORTATION NETWORK

DATASET

Many measures - including total number of passengers, total number of flights, or total amount of cargo - quantifying the importance of Brazilian airports are compiled and publicized in the *Agência Nacional de Aviação Civil* (ANAC) website². Data is organized in updated spreadsheets with information about authorized national and international flights³.

We restrict our analysis to passenger flights operating according to the data provided in October 20, 2013⁴. As flights do not have significant changes, we can assume that we are using a stable view of the airline network, based on the weekly information. At that time, there were 3, 579 national (interstate and intrastate flights) and 419 international flights, respectively. The total number of Brazilian airports is 120, 15 of them operating both national and international flights. A total number of 53 foreign airports have flights to/from Brazil. The total number of passengers on each flight is the maximum number of allowed passengers on each flight. We do not have access to the number of occupied seats for a specific flight. Therefore, here we present the upper bound values with respect to the maximum passenger capacity in each flight.

Some Brazilian airports found in the spreadsheets are shown in Table I. It shows the ICAO code⁵, the airport name and the city of the airport. An additional information⁶ was inserted in Table I to better analyze the air transportation network: the Instrument Landing System (ILS)⁷. This instrument helps pilots in landing operations and it is generally

used only when visibility is limited and the pilot cannot see the airport and the runway. The system is divided into three categories of approach from I to III. In “CAT I” ILS, for instance, pilot needs to have at least 200 ft of decision height and 1,600 ft of visibility. In “CAT IIIc” ILS, instead, pilot can land in any visibility condition. So far, Brazilian airports are not equipped with the safest ILS instrument. In this sense, Brazilian airports tend to be more vulnerable to bad weather conditions.

NETWORK MODELS

We focus our analysis on three scenarios that enable us to better characterize the Brazilian airport infrastructure, as well as to characterize the types of flights offered to the population. Although we present a more detailed analysis considering national connections, we also provide insights on international connections. Furthermore, we analyze a network model including both national and international connections. All metrics we used are usual metrics of complex network analysis (Newman 2003, Reka and Barabási 2002). We represented the Brazilian airport network as a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where \mathcal{V} is the set of airports and \mathcal{E} is the set of links. A link between two airports exists if there is at least one flight from one airport to another. Here, we consider two versions of graph \mathcal{G} : the unweighted and the weighted one. It is worth noting that we use the weighted version of each graph only for calculating the maximum number of passengers metric. (In this case, the maximum number of passengers is the link weight).

² <http://www2.anac.gov.br/hotran/>. Additional information can be found in <http://www.infraero.gov.br/index.php/situacao-dos-voos/por-aeroporto.html>

³ Stop flights were considered as a set of individual flights

⁴ 20th to 26th-October week

⁵ The International Civil Aviation Organization Code assigned to distinguish uniquely each airport

⁶ Obtained at <http://www.aisweb.aer.mil.br/?i=cartas&filtro=1&nova=1>

⁷ http://en.wikipedia.org/wiki/Instrument_landing_system

TABLE I
Main Brazilian Airports.

ICAO	Airport Name	City/State	ILS
SBGR	Governador André Franco Montoro	Guarulhos/SP	CAT II ^a
SBKP	Viracopos	Campinas/SP	CAT I
SBGL	Galeão - Antônio Carlos Jobim	Rio de Janeiro/RJ	CAT II
SBBR	Presidente Juscelino Kubitschek	Brasília/DF	CAT II
SBCF	Tancredo Neves	Belo Horizonte/MG	CAT I
SBSV	Deputado Luís Eduardo Magalhães	Salvador/BA	CAT I
SBPA	Salgado Filho	Porto Alegre/RS	CAT I
SBSP	Congonhas	São Paulo/SP	CAT I
SBCT	Afonso Pena	Curitiba/PR	CAT II
SBRF	Gilberto Freyre	Recife/PE	CAT I
SBCY	Marechal Rondon	Cuiabá/MT	CAT I
SBEG	Eduardo Gomes	Manaus/AM	CAT I
SBRJ	Santos Dumont	Rio de Janeiro/RJ	-
SBFZ	Pinto Martins	Fortaleza/CE	CAT I
SBBE	Val de Cães/Júlio Cezar Ribeiro	Belém/PA	CAT I
SBRP	Leite Lopes	Ribeirão Preto/SP	-
SBGO	Santa Genoveva	Goiânia/GO	-
SBCG	Campo Grande	Campo Grande/MS	CAT I
SBNT	Augusto Severo	Natal/RN	CAT I

^aGuarulhos has CAT IIIa installed since 2011 but to date is still not certified.

TABLE II
Number of nodes and edges for each connection network.

Scenario	Nodes	Edges
$\mathcal{G}_{national}$	120	726
$\mathcal{G}_{international}$	68	222
$\mathcal{G}_{overall}$	173	948

We consider three different network models: $\mathcal{G}_{national}$ with national flights, $\mathcal{G}_{international}$ with international flights and $\mathcal{G}_{overall}$, considering both national and international flights. Table II presents the total number of nodes (airports) and links (connections) on each graph.

NETWORK METRICS

We characterize the Brazilian air transportation network using classical graph theory metrics (a more detailed discussion on each metric can be found in (Newman 2003, Reka and Barabási 2002)). We used the interactive open source graph

visualization and manipulation platform software Gephi (Bastian et al. 2009) for rebuilding and analyzing the graph we modeled.

In-degree and out-degree

The *in-degree* of node v , $k_{in}(v)$, is the total number of incoming links. In the same way, the *out-degree* of node v , $k_{out}(v)$ is the total number of outgoing links. Then, the degree of node v , $k(v)$, is given by the summation of $k_{in}(v)$ and $k_{out}(v)$. The mean degree, $\langle k \rangle$, of a \mathcal{G} is given by:

$$\langle k \rangle (\mathcal{G}) = \sum_{\forall v \in \mathcal{V}} k(v) / |\mathcal{V}|.$$

Weighted in and out degrees are a straightforward definition of the unweighted version.

Average neighborhood overlap

Let $\mathcal{N}(u)$ and $\mathcal{N}(v)$ be the set of neighbors of nodes u and v , respectively. The neighborhood overlap, $n_o(u, v)$ of nodes u and v , is given by:

$$n_o(u, v) = (\mathcal{N}(u) \cap \mathcal{N}(v)) / (\mathcal{N}(u) \cup \mathcal{N}(v)),$$

also known as Jaccard Coefficient. Then, the average neighborhood overlap is given by:

$$\langle n_o \rangle = \frac{1}{|\mathcal{E}|} \sum_{\forall(u,v) \in \mathcal{E}} n_o(u,v).$$

Shortest path

Let $P_{u,v}$ be the set of paths between a given pair of nodes u and v . We define the shortest path $l(u,v)$ as the one having the lowest number of hops between source and destination. Let be also \mathcal{L} the set of all shortest paths $l(u,v), \forall(u,v)$. The mean shortest path $\langle l \rangle$, of a graph \mathcal{G} is given by:

$$\langle l \rangle (\mathcal{G}) = \frac{1}{|\mathcal{L}|} \sum_{\forall l(u,v) \in \mathcal{L}} l(u,v).$$

Diameter

Let $l(u,v)$ be the shortest path between nodes u and v . Diameter, \mathbf{d} , is defined as the longest shortest path between any pair of nodes in the network:

$$\mathbf{d}(\mathcal{G}) = \max_{\forall(u,v) \in \mathcal{V}} l(u,v).$$

Diameter property provides an idea of the dispersion in \mathcal{G} . In the air transportation network, diameter measure means the biggest trip in number of hops.

Clustering coefficient

We define the clustering coefficient, also known as network transitivity, as follows. In many networks, if node A is connected to node B and node B to node C, then there is a heightened probability that node A will also be connected to node C. In terms of network topology, transitivity means the presence of a heightened number of triangles in the network, i.e., sets of 3 nodes connected to each other. We define the clustering coefficient \mathcal{C} of \mathcal{G} as:

$$\mathcal{C}(\mathcal{G}) = \frac{3 \times \text{number of triangles in the network}}{\text{number of connected triples of vertices}}$$

Where a “connected triple” means a single node with edges running to an unordered pair of others.

Betweenness

The betweenness β_v of node v is the fraction of shortest paths connecting all pairs of nodes that pass through v . In other words, let $\sigma_{(j,k)}$ represent the number of shortest paths between nodes j and k , and $\sigma_{(j,k)}(v)$ the number of those paths that traverse node v . The betweenness of v is thus defined as:

$$\beta(v) = \sum_{j \neq v \neq k \in \mathcal{V}} \frac{\sigma_{(j,k)}(v)}{\sigma_{(j,k)}}.$$

Closeness

The closeness γ_v of node v captures how close it is from all other reachable nodes in the network. Given $\pi(v,k)$, the length of the shortest path between v and any other reachable node k , γ_v is defined as:

$$\gamma(v) = \left[\sum_{k \neq v, k \in \mathcal{V}} \pi(v,k) \right]^{-1}.$$

PageRank

PageRank is an algorithm used by the Google web search engine to rank websites in their search engine results. PageRank works by counting the number and quality of links to a node to determine a rough estimate of how important the node is. The underlying assumption is that more important nodes are likely to receive more links from other nodes (Brin and Page 1998). In the air transportation network it can assess which airport is more influential than others. The rank of a node P_i is given by the sum of all node ranks that point to node P_i divided by the number of nodes P_i points to:

$$r(P_i) = \sum_{P_j \in B_{P_i}} \frac{r(P_j)}{|P_j|},$$

$$r_{k+1}(P_i) = \sum_{P_j \in B_{P_i}} \frac{r_k(P_j)}{|P_j|}.$$

PageRank definition is recursive, and the process is iterated many times⁸.

⁸Results shown in Section 4 (See Air Transportation Network Analysis and Discussion) are normalized.

Graph density

The graph density D is defined as a ratio of the number of edges $|\mathcal{E}|$ to the number of possible edges (considering the complete graph):

$$D = 2|\mathcal{E}|/(|\mathcal{V}|(|\mathcal{V}| - 1)).$$

Connected component of the graph

A directed graph is strongly connected if every node is reachable from every other node. In this sense, the strongly connected components of an arbitrary directed graph form a partition into subgraphs that are themselves strongly connected.

TABLE III
Metrics of different Air Transportation Networks.

	Worldwide ^a	India ^b	China ^c	USA ^d	Italy ^e	BRAZIL ^f
Average Shortest Path	4.4	4	2.067	3.241	3.74	2.866
Average Clustering	0.62	0.6574	0.733	0.6208	0.1	0.451
Power Law Exponent	1.0	≈ 2.2	1.65	Partial 1.0512 ^g	0.2/1.7 ^h	1.0522

^a(Guimer et al. 2005), ^b(Bagler 2008), ^c(Li and Cai 2004), ^d(Cheung and Gunes 2012), ^e(Guida and Maria 2007), ^fOur results, ^gDegree distribution function is a mixed distribution, ^hDegree distribution is a combination of two power law distribution functions with coefficients equal to 0.2 and 1.7.

AIR TRANSPORTATION NETWORK ANALYSIS AND DISCUSSION

This section describes the results obtained from the analysis of our database. First, the Brazilian air transportation network is compared with other country's airplane networks previously analyzed in the literature. Then, a more detailed analysis about the national flights is presented indicating some characteristics of the Brazilian air transportation network. Moreover, we show the community formation and the resiliency analysis of the Brazilian network. Finally the International and the Overall view of the Brazilian airports network is presented showing some peculiarities of the network, such as to which international airports Brazil is connected.

BRAZILIAN AIR TRANSPORTATION NETWORK VERSUS FOREIGN AIR TRANSPORTATION NETWORKS

Table III shows the comparison of Brazilian airline network against the set of foreign networks found in literature (Guimer et al. 2005, Bagler 2008, Li and Cai 2004, Cheung and Gunes 2012, Guida and Maria 2007). To provide a better comparison among the characteristics of different airline networks, Table IV shows some indicators of each country we show in Table III, considering the year of the dataset analyzed in each work found in literature.

First we can note that node degree on the $\mathcal{G}_{national}$ follows a power law, which means that there are few airports with many connections and many airports with few connections. Such network, whose degree

TABLE IV
Some important indicators of each country.

Country	Population	Area (km ²)	Airports	Gross Domestic Product
India	1.110 bi	2.973 mi	79	US\$721.585 bi
China	1.296 bi	9.327 mi	128	US\$ 1.931 tri
USA	311.587 mi	9.147 mi	850	US\$ 15.533 tri
Italy	58.94 mi	294,140	42	US\$ 1.873 tri
Brazil	201.032 mi	8.459 mi	120	US\$2.242 tri

distribution follows a power law, is called scale-free network. Second, the average shortest path is also low, with few hops to move from place to place. Furthermore, the Brazilian network has lower average clustering value, in comparison to the others. This implies that there are fewer triangles in the network, leading to a low redundancy in paths between airports. From indicators showed in Table IV, we can note that Brazil has a larger number of airports per capita than India and China. USA, as expected, is the country with the most airports, mainly because of its area and its economic power. However, considering only the topological metrics, results reveal that the Brazilian airline network follows a similar structure when compared to other airlines network.

NATIONAL NETWORK CHARACTERISTICS

Next, we consider the 3,579 national flights operated by the airline companies as well as the 120 airports in which flights take off from and land at. In this sense, the national graph version of \mathcal{G} has 120 nodes and 726 links. The maximum number of passengers that can move per week is equal to 2, 631, 836. In other words, this number is the total number of passengers in the case that all flights were completely full. Table V shows the global metrics. We found some interesting conclusions. The generated graph is connected, meaning that it is possible to reach all airports from another. Airport connectivity is low: on average, each airport connects to another 6 airports (5% of total number of nodes). In the United States, for instance, the mean of connections is 24 airports but it only represents 2.8% of the total number of airports. Furthermore, the average neighborhood overlap is equal to 0.163. Edges with very small neighborhood overlap act as local bridges, since intuitively, edges with very small neighborhood consist of nodes that span over different regions of the graph. In this sense, the value of neighborhood overlap of the Brazilian airline infrastructure reveals that the network is composed by many airports that act as bridges. This kind of infrastructure directly

impacts on the network connectivity: a removal of an edge can make an increasing on the number of trip hops to a particular destination or, even worst, to disconnect the network.

TABLE V
Network metrics - National Flights.

Network Measure	Value
Connected Components	1 (120 airports)
Average Connections	6.05 (5%)
Average Weighted Degree	21,932
Diameter	7
Average Shortest Path	2.866
Graph Density	0.051
Average Clustering Coefficient	0.451
Average Neighborhood Overlap	0.163

Passengers in Brazil take, on average, 3 flights to move from place to place (the average shortest path is equal to 2.86). Interestingly, the largest trip in Brazil has 7 flights (diameter graph) and is the trip from Confresa Airport (state of Mato Grosso) to Pato de Minas Airport (state of Minas Gerais) (cities are just 1,472 km apart from each other). Finally, the average clustering coefficient is 0.451. This implies that an airport has 45% of chance to be connected to another airport in the network. Lastly, the maximum number of passengers that can fly every week is, on average, equal to 21, 932 per airport.

To provide a deeper analysis, we also investigate the local characteristics of the network structure. The local analysis allows us to identify the main airports in Brazil and their roles in the overall operation of the air transportation network. 93.3% of the airports have less than 40 connections. Just 8 airports have more than 40 connections. The airport with the most connections from/to is Viracopos in Campinas city (SBKP) with 105. It is clear that the network infrastructure follows a scale-free model, with some airports acting like hubs.

An interesting finding is that Viracopos Airport plays an important role in the national network infrastructure. This affirmation is corroborated by the top-20 most central airports, considering the

TABLE VI
Top-20 Airports - Number of Connections - National Flights.

Name (ICAO)	Connections	Passengers	Betweenness	PageRank
Viracopos (SBKP)	105	256,234	0.3	0.06
Guarulhos (SBGR)	86	602,801	0.15	0.04
Brasília (SBBR)	74	450,647	0.2	0.04
Confins (SBCF)	59	280,340	0.09	0.03
Congonhas (SBSP)	52	530,779	0.03	0.02
Galeão (SBGL)	48	331,433	0.03	0.02
Salvador (SBSV)	46	230,771	0.05	0.02
Porto Alegre (SBPA)	41	197,394	0.08	0.02
Cuiabá (SBCY)	38	81,963	0.08	0.02
Curitiba (SBCT)	36	216,513	0.04	0.02
Manaus (SBEG)	34	313,519	0.03	0.01
Santos Dumont (SBRJ)	34	82,937	0.22	0.02
Belém (SBBE)	30	177,179	0.01	0.01
Recife (SBRF)	30	118,535	0.08	0.02
Fortaleza (SBFZ)	28	163,117	0	0.01
Ribeirão Preto (SBRP)	24	34,852	0	0.01
Goiânia (SBGO)	23	91,689	0.01	0.01
Campo Grande (SBCG)	22	45,563	0.01	0.01
Pampulha (SBBH)	21	16,838	0.05	0.01
Vitória (SBVT)	20	95,236	0	0.01

main centrality measures, shown in Tables VI to IX (Airport label is its ICAO⁹ code). Although Viracopos Airport does not hold the maximum number of passengers per week (Guarulhos Airport has the maximum number of passengers), it is the most frequent airport present in the shortest paths over the network (high betweenness centrality). Furthermore, considering the number of passengers and the airports that have at least one flight between them, Viracopos has also the highest PageRank centrality. We conjecture that Viracopos turns to one the most important Brazilian airports after the Azul Airline Company¹⁰ creation. Viracopos airport is the main company hub and a large number of routes pass through it, increasing a lot the importance of this airport in the national connections network.

Figure 1 shows a big picture of Brazilian airline network. The node size is proportional to its degree

and the color to its betweenness centrality (red color means high betweenness centrality). Some interesting conclusions can be hold. The most important airports are not well spread over the Brazilian territory. The most connected airports are concentrated in the Southeast region. In some cases, people from North and Northeast regions need fly to some hub in the Southeast region in order to go back to some city in those regions. The only airport in North and Northeast region that plays an important role in Brazil airline infrastructure is Manaus Airport (SBEG). The majority of airports have few connections and are not placed into the shortest paths in the airline network (the blue ones). As expected, the most important airports are placed in the capital cities.

Edges in Figure 1 represent the total number of passengers between two airports and their thicknesses are proportional to the total number of passengers on the route. The total number of passengers between Congonhas and Santos Dumont Airports (148, 352 passengers per week) is twice the second busiest graph connection, i.e., Congonhas and Brasília Airports (60, 129 passengers per week). The route

⁹<http://www.icao.int/Pages/default.aspx>

¹⁰<http://www.voeazul.com.br/>

TABLE VII
Top-20 Airports - Maximum Number
of Passengers - National Flights.

Name (ICAO)	Passengers	Connections	Betweenness	PageRank
Guarulhos (SBGR)	602,801	86	0.15	0.04
Congonhas (SBSP)	530,779	52	0.03	0.02
Brasília (SBBR)	450,647	74	0.2	0.04
Galeão (SBGL)	331,433	48	0.03	0.02
Santos Dumont (SBRJ)	313,519	34	0.03	0.01
Confins (SBCF)	280,340	59	0.09	0.03
Viracopos (SBKP)	256,234	105	0.3	0.06
Salvador (SBSV)	230,771	46	0.05	0.02
Curitiba (SBCT)	216,513	36	0.04	0.02
Porto Alegre (SBPA)	197,394	41	0.08	0.02
Recife (SBRF)	177,179	30	0.01	0.01
Fortaleza (SBFZ)	163,117	28	0	0.01
Belém (SBBE)	118,535	30	0.08	0.02
Florianópolis (SBFL)	99,775	14	0	0
Vitória (SBVT)	95,236	20	0	0.01
Goiânia (SBGO)	91,689	23	0.01	0.01
Manaus (SBEG)	82,937	34	0.22	0.02
Cuiabá (SBCY)	81,963	38	0.08	0.02
Natal (SBNT)	59,680	17	0	0
São Luís (SBSL)	58,170	16	0	0

TABLE VIII
Top-20 Airports - Betweenness
Centrality - National Flights.

Name (ICAO)	Betweenness	Connections	Passengers	PageRank
Viracopos (SBKP)	0.30	105	256,234	0.06
Manaus (SBEG)	0.22	34	82,937	0.02
Brasília (SBBR)	0.20	74	450,647	0.04
Guarulhos (SBGR)	0.15	86	602,801	0.04
Confins (SBCF)	0.09	59	280,340	0.03
Belém (SBBE)	0.08	30	118,535	0.02
Cuiabá (SBCY)	0.08	38	81,963	0.02
Tefé (SBTF)	0.08	14	3,523	0.02
Porto Alegre (SBPA)	0.08	41	197,394	0.02
Palmas (SBPJ)	0.05	12	18,714	0.01
Salvador (SBSV)	0.05	46	230,771	0.02
Pampulha (SBBH)	0.05	21	16,838	0.01
Curitiba (SBCT)	0.04	36	216,513	0.02
Santos Dumont (SBRJ)	0.03	34	313,519	0.01
Santarém (SBSN)	0.03	10	25,352	0.01
Congonhas (SBSP)	0.03	52	530,779	0.02
Galeão (SBGL)	0.03	48	331,433	0.02
Marabá (SBMA)	0.02	14	18,286	0.01
Campo Grande (SBCG)	0.01	22	45,563	0.01
Goiânia (SBGO)	0.01	23	91,689	0.01

TABLE IX
Top-20 Airports - PageRank
Centrality - National Flights.

Name (ICAO)	PageRank	Connections	Passengers	Betweenness
Viracopos (SBKP)	0.06	105	256,234	0.30
Guarulhos (SBGR)	0.04	86	602,801	0.15
Brasília (SBBR)	0.04	74	450,647	0.20
Confins (SBCF)	0.03	59	280,340	0.09
Manaus (SBEG)	0.02	34	82,937	0.22
Congonhas (SBSP)	0.02	52	530,779	0.03
Salvador (SBSV)	0.02	46	230,771	0.05
Porto Alegre (SBPA)	0.02	41	197,394	0.08
Galeão (SBGL)	0.02	48	331,433	0.03
Cuiabá (SBCY)	0.02	38	81,963	0.08
Tefé (SBTF)	0.02	14	3,523	0.08
Belém (SBBE)	0.02	30	118,535	0.08
Curitiba (SBCT)	0.02	36	216,513	0.04
Santos Dumont (SBRJ)	0.01	34	313,519	0.03
Pampulha (SBBH)	0.01	21	16,838	0.05
Recife (SBRF)	0.01	30	177,179	0.01
Fortaleza (SBFZ)	0.01	28	163,117	0.01
Goiânia (SBGO)	0.01	23	91,689	0.01
Ribeirão Preto (SBRP)	0.01	24	34,852	0.00
Campo Grande (SBCG)	0.01	22	45,563	0.01

community found by the algorithm. Interestingly, the communities structure almost reflects the Brazilian regions. North (Amazonia) region has the Manaus Airport (SBEG) as the major one. We also have a smaller group (pink one) that represents some airports of states of Tocantins, Pará and Mato Grosso, such as Marabá Airport (SBMA) and Palmas Airport (SBPJ) each with less than 19,000 passengers per week. The remaining airports are small ones with less than 4,000 passengers per week.

The green community is very similar, geographically speaking, to the Northeast Brazilian region. However, this community also includes Brasília, Confins and Galeão airports, meaning that the Northeast region is highly dependant on the South Airports. The dark blue community is composed by small airports, mainly from the state of Minas Gerais. The light blue community has Viracopos Airport, the most connected airport considering national flight connections. Furthermore, it englobes many airports from the Southeast region. The yellow

TABLE X
Top-20 Airports - Highest Closeness
Centrality - National Flights.

Name (ICAO)	Closeness(Average Shortest Path)	Connections
Viracopos (SBKP)	0.58 (1.7)	105
Brasília (SBBR)	0.55 (1.8)	74
Guarulhos (SBGR)	0.54 (1.84)	86
Confins (SBCF)	0.52 (1.91)	59
Galeão (SBGL)	0.5 (1.98)	48
Manaus (SBEG)	0.46 (2.13)	34
Belém (SBBE)	0.45 (2.17)	30
Fortaleza (SBFZ)	0.45 (2.18)	28
Congonhas (SBSP)	0.45 (2.2)	52
Curitiba (SBCT)	0.45 (2.21)	36
Salvador (SBSV)	0.44 (2.22)	46
Porto Alegre (SBPA)	0.44 (2.22)	41
Santos Dumont (SBRJ)	0.44 (2.24)	34
Cuiabá (SBCY)	0.44 (2.26)	38
Goiânia (SBGO)	0.43 (2.3)	23
Recife (SBRF)	0.42 (2.33)	30
Ribeirão Preto (SBRP)	0.42 (2.35)	24
Campo Grande (SBCG)	0.42 (2.35)	22
Vitria (SBVT)	0.42 (2.36)	20
Aracaju (SBAR)	0.42 (2.36)	18

community is composed by airports from the South region as well as by the Guarulhos and Congonhas airports. The community analysis provides a nice way of identifying airports dependencies from both structural and economic points of view. Moreover, it is also possible to have some insights of air company's economics interests.

Resiliency analysis

Resiliency analysis gives important insights on the airline network robustness under topology changes. For instance, some airports can be closed as a consequence of bad weather conditions or operational problems. Consequently, routes have to be redefined. Here, we study the impact on the number of components as well as on the total number of passengers when some airports are removed from the network.

Our resiliency analysis proceeds as follows. We perform airports removals targeting the most central nodes in the network, according to the number of connections, betweenness and number of passengers. We remove nodes in decreasing order

TABLE XI
The 20 airports with the lowest closeness centrality values.

Name (ICAO)	Closeness	Connections
Confresa (SJHG)	0.21 (4.65)	4
Umarama (SSUM)	0.23 (4.17)	2
Tapuruquara (SWTP)	0.24 (4.02)	2
São Paulo de Olivença (SDCG)	0.24 (4.02)	2
Fonte Boa (SWOB)	0.24 (4.02)	2
Eirunepé (SWEI)	0.24 (4.02)	2
Uaupés (SBUA)	0.24 (4.02)	2
Porto Trombetas (SBTB)	0.25 (3.99)	2
Itaituba (SBIH)	0.25 (3.99)	2
Cruzeiro do Sul (SBCZ)	0.26 (3.75)	2
Vila Rica (SWVC)	0.27 (3.68)	4
São Félix do Araguaia (SWFX)	0.27 (3.68)	4
São João del-Rei (SNJR)	0.27 (3.66)	2
Ourilândia do Norte (SDOW)	0.27 (3.57)	4
Redenção (SNDC)	0.27 (3.57)	4
Erechim (SSER)	0.28 (3.55)	2
Governador Valadares (SBGV)	0.28 (3.51)	3
Patos de Minas (SNPD)	0.28 (3.50)	2
Araxá (SBAX)	0.28 (3.49)	3
Corumbá (SBCR)	0.29 (3.34)	2

of their metric values. Consecutive removals are performed until the giant component achieves half of its initial size.

Figures 3 and 4 show the results. Considering the number of components, the worst case happens when airports with the highest betweenness measures are removed from the network. By removing the three airports with the highest betweenness (Viracopos, Guarulhos and Brasília in this order), the network is fragmented into 6 connected components. Considering the percentage of the passengers, the worst case occurs by removing four airports (Guarulhos, Congonhas, Brasília and Galeão Airports in this order). The total number of passengers drops to almost 30% of the total capacity.

This analysis shows how dangerous it is to remove an airport in the network. For instance, let us consider the Viracopos airport (SBKP). It is the most important airport considering national connections, in terms of topological characteristics. As expected, its removal can cause many disconnection points. Table I shows that Viracopos airport is equipped with ILS CAT I which is not suitable for dealing with extreme weather conditions. Then, Viracopos airport has high chances of being closed due to bad weather conditions. This fact influences the network topology stability.

Time analysis

The results in the previous sections do not take into account any information about flight duration and flight daily distribution. To investigate the time impact in our analysis, we included the flight duration in each edge of our graph model. Including time in our analysis allows to obtain some interesting results.

Figure 5 shows the air transportation network where nodes are proportional to the time flights among airports: nodes with larger size represent airports with greater average flight durations from them. The largest one-hop flight departs from Galeão Airport (SBGL) and arrives at

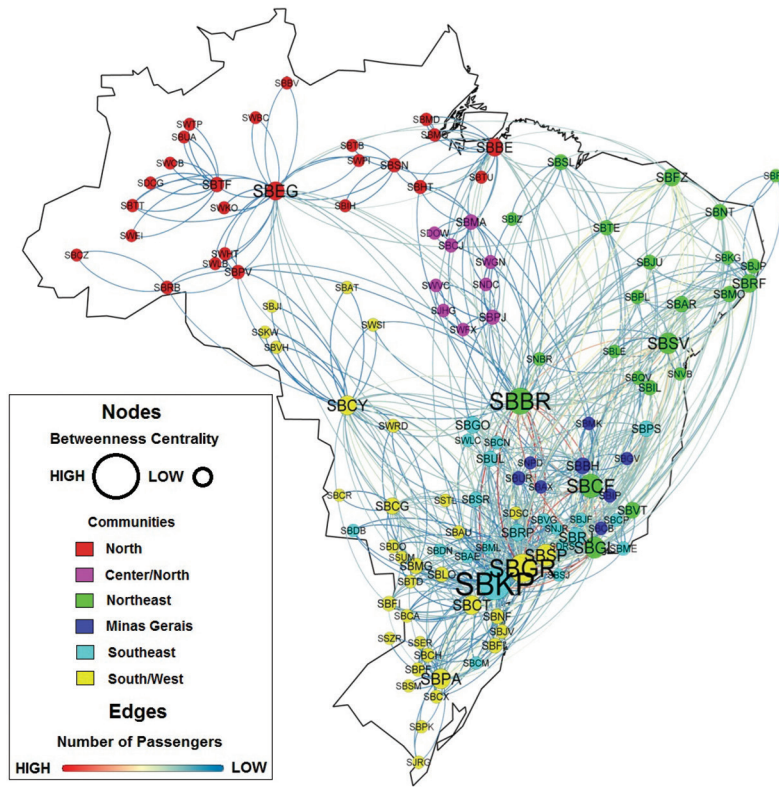


Figure 2 - Airports grouped by different communities. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

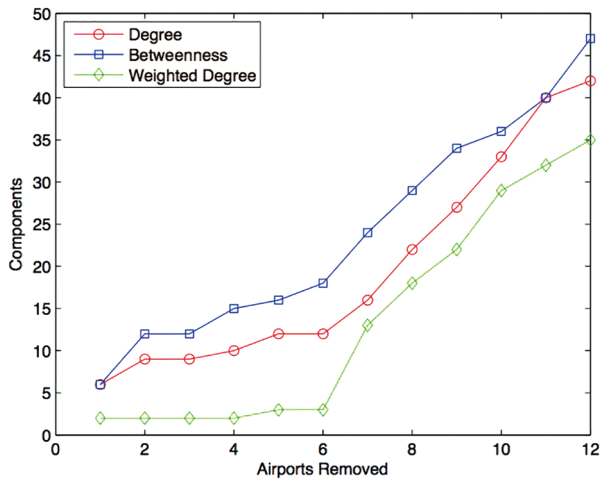


Figure 3 - Changes to the number of components by the total number of removed airports. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

Manaus Airport (SBEG) with duration of 245 minutes. The smallest one-hop flight departs from Ipatinga Airport (SBIP) and arrives at Governador

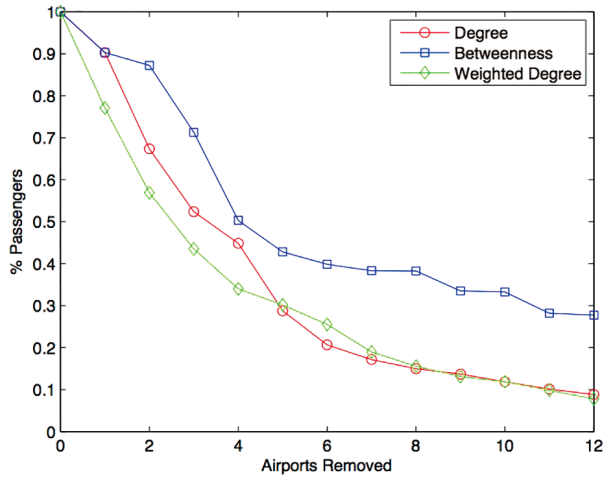


Figure 4 - Changes to the percentage of passengers by the total number of removed airports. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

Valadares Airport (SBGV) with duration of 15 minutes. The results corroborate the metrics previously calculated.

TABLE XII
Top-20 largest average time of all paths
in the air transportation network.

ICAO	Average Travel Time (min)	Longest Travel Time (min)
SWEI	441.8	617
SBUA	417.01	592
SDCG	417.01	592
SBTT	396.54	570
SWTP	392.22	567
SBCZ	384.72	525
SWOB	382.31	557
SWLB	365.75	533
SBTB	358.54	551
SBJI	355.9	513
SBBV	353.79	521
SBIH	352.52	545
SSZR	343.63	611
SBTF	337.68	512
SWKO	334.06	501
SWBC	333.96	501
SWPI	331	498
SJHG	326.54	521
SBFN	326.3	501
SSUM	325.99	593

that have flights to Brazil. Graph $\mathcal{G}_{international}$ has 68 airports, 15 of them in Brazil and 53 of them overseas.

Different from the national view, the generated graph has two components. One englobes Belém (SBBE), Surinam and French Guiana airports. The other component is the giant strongly connected component covering all other Brazilian and foreign airports. The main global metrics are summarized in Table XIV on average, every international Brazilian airport has connections to 3 other foreign airports. Approximately 7, 208 passengers travel per airport in a week.

Figure 7 shows the two main graph components. Furthermore, node size is proportional to the connections to a given airport: Guarulhos Airport (SBGR) plays the most important role in the airline international network, followed by the Galeão Airport (SBGL). Table XV shows the total number of international passengers supported by each airport. Guarulhos and Galeão Airports hold, respectively, 313,275 and 100,886 passengers per week (84% of the international passengers). This result reinforces the importance

TABLE XIII
Top-20 smallest average time of all paths
in the air transportation network.

Name (ICAO)	Average Travel Time (min)	Longest Travel Time (min)
Brasília (SBBR)	178.63	355
Viracopos (SBKP)	182.16	409
Confins (SBCF)	185.23	391
Guarulhos (SBGR)	190.21	408
Goiânia (SBGO)	197	395
Congonhas (SBSP)	199.71	458
Galeão (SBGL)	203.86	417
Uberlândia (SBUL)	204.05	425
Santos Dumont (SBRJ)	206.52	458
Curitiba (SBCT)	208.51	473
Ribeirão Preto (SBRP)	210.56	460
Cuiabá (SBCY)	217.06	373
Caldas Novas (SBCN)	217.86	432
Vitória (SBVT)	222.09	457
Navegantes (SBNF)	222.77	474
Londrina (SBLO)	224.28	472
São José R.P (SBSR)	224.34	473
Campo Grande (SBCG)	227.66	446
Ipatinga (SBIP)	227.75	439
Porto Seguro (SBPS)	229.91	466

TABLE XIV
International Connections.

Metric	Value
Connected Components	2 (68 airports)
Average Connections	3.265
Average Weighted Degree	7, 208
Diameter	6
Average Shortest Path	2.389
Graph Density	0.049

of Guarulhos and Galeão airports and the need for efficient contingency policies in both airports.

Figure 8 shows that Brazil has three main overseas connections: Miami (KMIA), Buenos Aires (SAEZ) and Lisboa Airports (LPPT). Airports in Brazil are the main en trances/exits to/from South America of people coming to/from North America

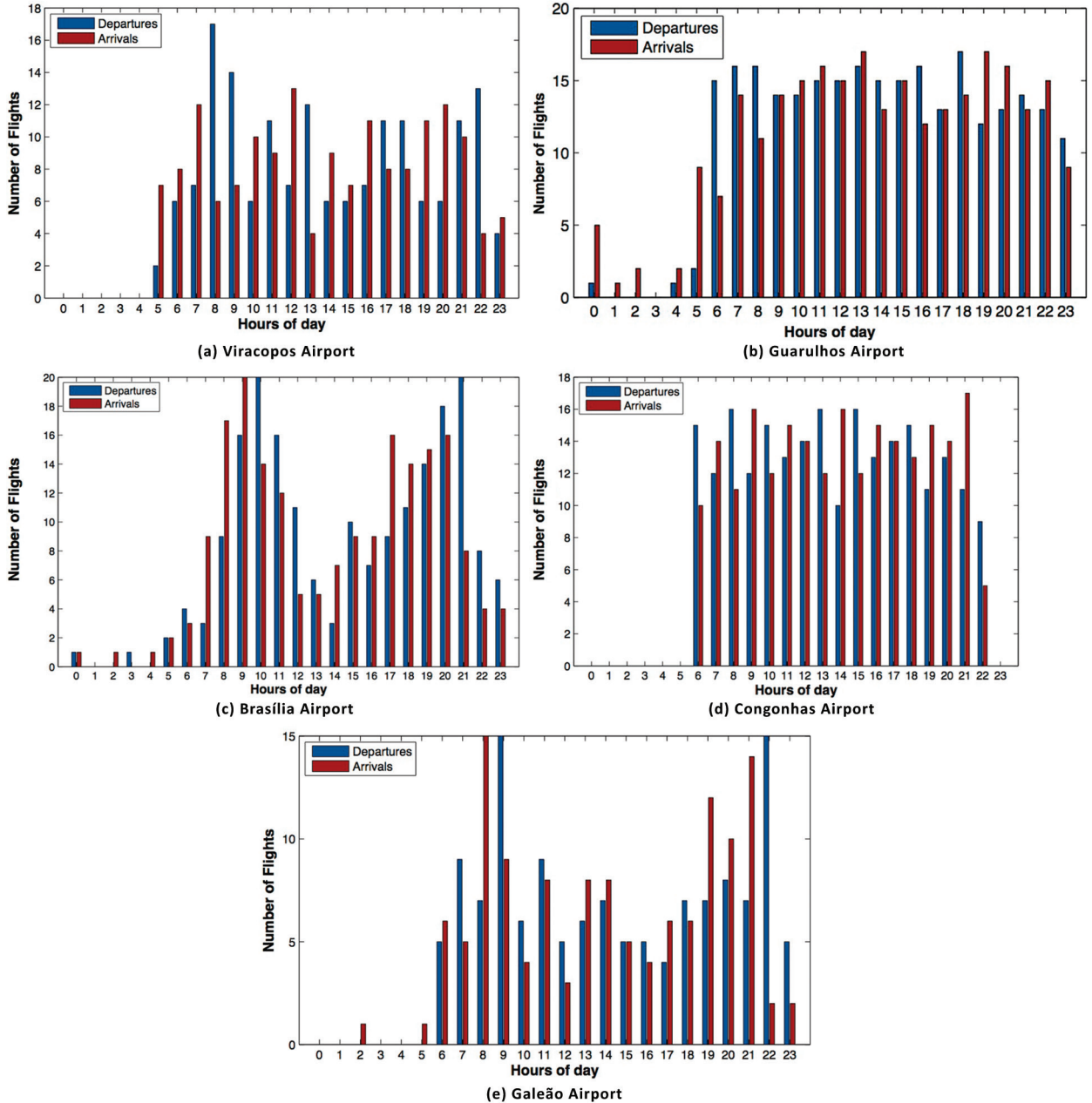


Figure 6 - Daily distribution of flights. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

TABLE XV
Top-20 - Maximum Number of
Passengers - International Flights.

Name (ICAO)	Passengers	Connections	Passengers (in)	Passengers (out)
Guarulhos (SBGR)	313,275	96	156,774	156,501
Galeão (SBGL)	100,886	50	50,443	50,443
Ezeiza (SAEZ)	52,260	16	26,130	26,130
Miami (KMIA)	44,824	14	22,412	22,412
Lisbon (LPPT)	36,868	20	18,434	18,434
Santiago (SCEL)	26,714	4	13,357	13,357
Jorge Newbery (SABE)	25,270	6	12,635	12,635
John F. Kennedy (KJFK)	22,892	4	11,446	11,446
Tocumen (MPTO)	21,964	14	10,982	10,982
Charles de Gaulle (LFPG)	19,366	4	9,683	9,683
Madrid-Barajas (LEMD)	18,544	6	9,272	9,272
Carrasco (SUMU)	15,501	6	7,614	7,887
Frankfurt (EDDF)	15,088	8	7,544	7,544
London Heathrow (EGLL)	14,472	4	7,236	7,236
Jorge Chvez (SPIM)	13,882	8	6,941	6,941
Porto Alegre (SBPA)	13,702	12	6,851	6,851
Brasília (SBBR)	13,662	10	6,831	6,831
El Dorado (SKBO)	9,982	4	4,991	4,991
Confins (SBCF)	9,838	8	4,919	4,919

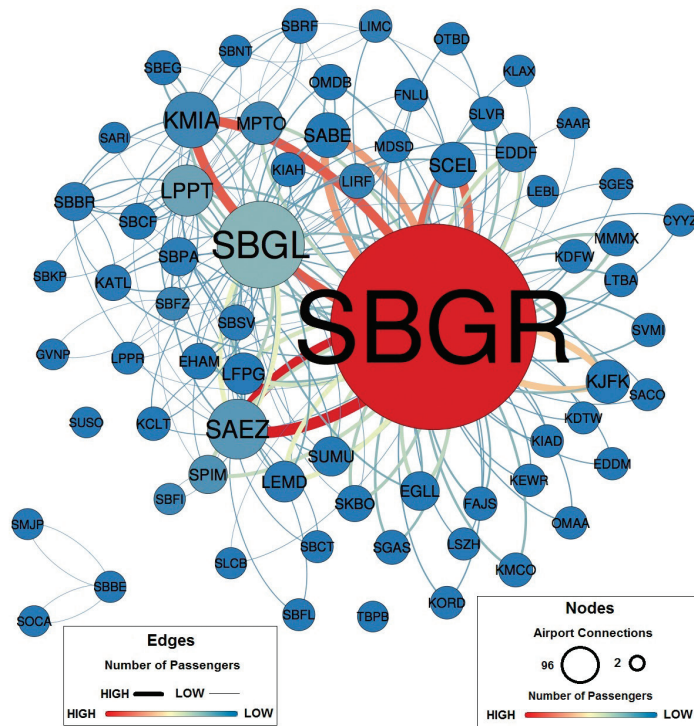


Figure 7 - Network - International Flights. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

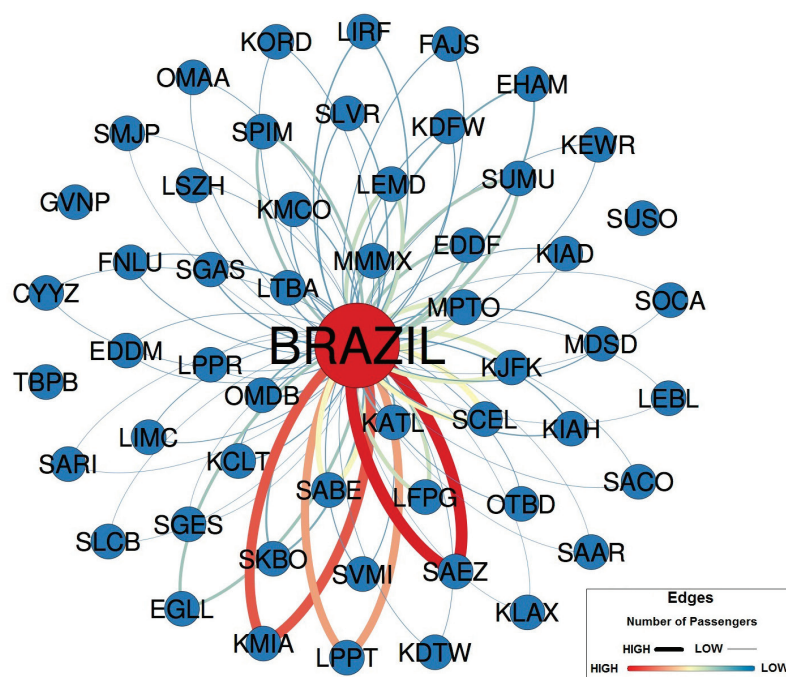


Figure 8 - Brazilian Connections from/to Worldwide. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

and Europe. Considering the current flights, the maximum number of passengers that are allowed to come to Brazil and to exit from Brazil are 245,232 and 244,959 passengers per week, respectively.

In this section, our analysis considers the national and international connections from/to Brazilian airports. $G_{overall}$ graph has one component and, in the average, each airport connects to another 5.48 airports. Graph density is low, meaning that the connections are too sparse. On average, the

total number of passengers is equal to 18,046. The average trip size is equal to 2.76. Table XVI summarizes the results.

Considering both national and international connections, Guarulhos and Viracopos are the most central airports in Brazil, as expected. Guarulhos plays a key role in airline network, dealing with the largest number of passengers and flights. Table XVII shows the main centrality measures, considering the top-20 Brazilian airports for the number of connections. As expected, the most connected airports are the state capitals ones and all of them have high closeness centrality.

Resiliency analysis

In this section we briefly discuss the resiliency of the international network. Our analysis proceed very similarly to the discussion in Section 4.2.2. (See Resiliency Analysis) As for the national network, we perform airports removals targeting the most central nodes in the network, according to the number of connections, betweenness and number of

TABLE XVI
National and International Connections.

Metric	Value
Connected Components	1 (173 airports)
Average Connections	5.48 (3.16%)
Average Weighted Degree	18, 046
Diameter	7
Average Shortest Path	2.76
Graph Density	0.03
Average Clustering Coefficient	0.44
Average Neighborhood Overlap	0.12

TABLE XVII
Top-20 Brazilian Airports - Number of Connections -
National and International Flights.

Name (ICAO)	Connections	Passengers	Closeness	Betweenness	PageRank
Guarulhos (SBGR)	182	916,076	0.61	0.44	0.095
Viracopos (SBKP)	107	257,812	0.51	0.18	0.048
Galeão (SBGL)	98	432,319	0.54	0.1	0.044
Brasília (SBBR)	84	464,309	0.54	0.14	0.035
Confins (SBCF)	67	290,178	0.51	0.05	0.028
Salvador (SBSV)	56	239,251	0.46	0.04	0.023
Porto Alegre (SBPA)	53	211,096	0.46	0.06	0.024
Congonhas (SBSP)	52	530,779	0.43	0.02	0.021
Recife (SBRF)	38	184,673	0.44	0.01	0.015
Cuiabá (SBCY)	38	81,963	0.45	0.05	0.019
Curitiba (SBCT)	38	219,075	0.46	0.03	0.016
Manaus (SBEG)	38	89,475	0.47	0.16	0.022
Santos Dumont (SBRJ)	34	313,519	0.45	0.02	0.014
Belém (SBBE)	34	119,883	0.47	0.08	0.018
Fortaleza (SBFZ)	34	167,800	0.47	0.01	0.014
Ribeirão Preto (SBRP)	24	34,852	0.44	0	0.01
Goiânia (SBGO)	23	91,689	0.44	0.01	0.01
Campo Grande (SBCG)	22	45,563	0.44	0.01	0.01
Pampulha (SBBH)	21	16,838	0.41	0.03	0.012
Natal (SBNT)	21	61,839	0.43	0	0.008

passengers. Furthermore, we only remove Brazilian Airports. Figures 9 and 10 show the results.

From this simple analysis, it is possible to corroborate the importance of Guarulhos and

Galeão Airports as exit point from Brazil to other countries. For all centrality metrics analyzed, both airports were removed in the first and second places. For the cases which Guarulhos Airport is

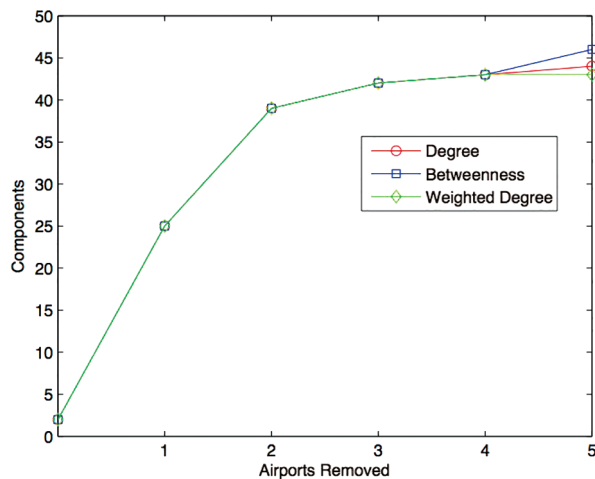


Figure 9 - Changes on the number of components by the total number of removed airports. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

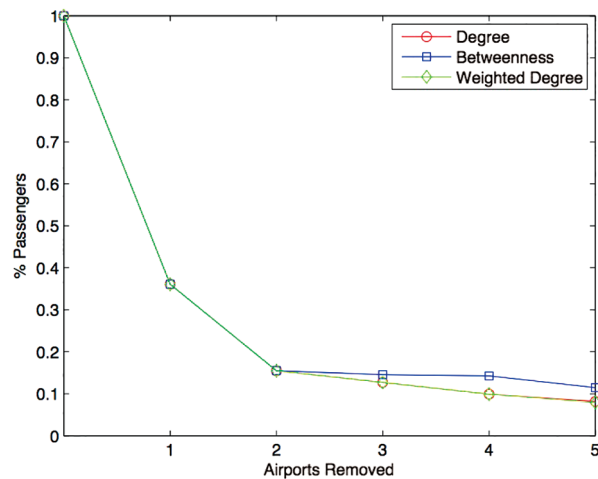


Figure 10 - Changes on the percentage of passengers by the total number of removed airports. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

removed from the network, only 36% of passengers are allowed to travel abroad. Putting together the Galeão Airport, this number drops to 15%. A similar impact can also be seen on the increase of the number of components in the network.

CONCLUSIONS

In this paper we analyzed the main topological characteristics of the Brazilian air transportation network, based on the set of national and international flights operated by the Brazilian airports.

The Brazilian network has small world properties and the airport connections follow a power law distribution. Our results showed that the main airports in the Brazilian infrastructure are the Viracopos and Guarulhos airports. Furthermore, travelers need, on average, 3 connection flights to reach their destinations. We also performed the resiliency analysis of the network robustness under topology changes. We showed that the Viracopos Airport outage breaks the network into 6 subnetworks, affecting 10% of the passengers.

Some interesting analysis can be performed based on the results discussed in this work. For instance, it is important to know the impact of closing an airport, for a given amount of hours. Furthermore, it is also interesting to have some insights on how long the transfers are. We can also analyze the ticket prices across the Brazilian regions. We plan to address these issues next.

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RESUMO

A rede de transporte aéreo em um país tem um grande impacto na economia local, nacional e global. Neste artigo, analisamos a rede de transporte aéreo no Brasil através de métricas de redes complexas para o melhor entendimento de suas características. Em nossa análise, definimos redes compostas por voos nacionais ou internacionais. Também consideramos a rede onde ambos os tipos de voos são reunidos. Conclusões interessantes emergem da nossa análise. Por exemplo, o Aeroporto de Viracopos (cidade de Campinas) é o aeroporto mais central e conectado na rede de voos nacionais. Qualquer problema operacional neste aeroporto separa a rede brasileira em 6 subredes distintas. Mais ainda, a rede de conexão aérea nacional brasileira possui características de mundo pequeno e a rede das conexões nacionais segue uma distribuição de lei de potência. Portanto, nossa análise possibilita compreender a infraestrutura da rede de transporte aéreo Brasileira, trazendo um novo entendimento para ajudar a lidar com o rápido crescimento recente do uso da rede de transporte aéreo no Brasil.

Palavras-chave: Rede brasileira de Transporte Aéreo, ciência de redes, análise da estrutura da rede, redes complexas.

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